

# Experimental Evidence on the Persistence of Output and Inflation\*

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## Abstract

This paper presents experimental evidence from a monetary sticky price economy in which output and inflation depend on expected future inflation. Rational inflation expectations do not allow for persistent deviations of output and inflation following a monetary shock. In the experimental sessions, however, output and inflation display considerable persistence and regular cyclical patterns. This emerges because subjects' inflation expectations fail to be captured by rational expectations functions. Instead, a Restricted Perceptions Equilibrium (RPE), which assumes that agents use optimal but 'simple' forecast functions, describes subjects' inflation expectations surprisingly well and explains the observed behavior of output and inflation.

KEYWORDS: Experiments, Output and Inflation Dynamics, Restricted Perceptions Equilibrium, Rational Expectations

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# 1 Introduction

Rational expectations models with nominal rigidities, workhorses of current macroeconomics, seem to have rather weak internal propagation mechanisms and therefore face difficulties in matching the persistence inherent in output and inflation data. Especially, matching the reactions of output and inflation in response to nominal shocks has proven cumbersome (e.g. Caplin and Spulber (1987), Chari et al. (2000), or Nelson (1998)).

An important literature suggests that this difficulty is the result of overly simplistic models, i.e., lack of major frictions, and can be overcome by modelling these frictions, e.g., Smets and Wouters (2003) or Christiano et al. (2005). At the same time, it has been suggested that implausibly strong rationality assumptions may be the source of the persistence problem. In particular, fairly *simple* economic models have been shown to display strong internal propagation once the assumption of rational expectations is relaxed, e.g., Evans and Ramey (1992), Evans and Honkapohja (1993),(2001), Sargent (1999), or Adam (2005b).

This paper studies a simple monetary sticky price economy with nominal demand shocks in the experimental laboratory and assesses whether deviations of expectations from rational expectations are an important factor contributing to the persistence of output and inflation. Importantly, the paper presents direct evidence on the relevance of non-rationalities and their implications for the persistence of output and inflation.

Resorting to laboratory experiments is justified on the grounds that expectations in the field are generally not easily observed.<sup>1</sup> This makes it difficult to identify deviations from rational expectations. Moreover, without reference to a specific model, the economic relevance of identified deviations cannot be interpreted but reaching consensus about the relevant economic model seems difficult. Laboratory experiments do not face such problems because agents' expectations can be made directly observable and the economic model is true by definition.

The monetary economy studied in this paper is extremely simple: a cash-in-advance constraint forces agents to hold money in equilibrium and sticky prices cause nominal demand shocks to have real effects. Since prices are preset for one period only, the model displays 'New Classical' features under rational expectations, i.e., output reacts to unexpected money shocks only and inflation lags output by one period. As a result, the model does not generate persistent deviations of output and inflation in response to a

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<sup>1</sup>Survey expectations provide some evidence on expectations and have been used in Roberts (1995), Carroll (2003), or Adam and Padula (2003), amongst others.

monetary shock, thus, displays 'weak internal propagation' under rational expectations.

More generally, however, the model's temporary equilibrium equation implies that current output and inflation depend on lagged endogenous variables, agents' expectations of future inflation rates, and exogenous nominal demand shocks.<sup>2</sup> Thus, whenever expectations deviate from rational ones, the behavior of output and inflation also deviates from the white noise prediction.

To test for the relevance of such deviations, subjects in the experiments are asked to predict future inflation rates. Subjects have an incentive to provide the best possible forecast because the payments they receive depend negatively on their forecast errors. Subjects' inflation forecasts are then substituted into the model's temporary equilibrium equation and determine a model-consistent outcome for output and inflation. The new output level and inflation rate is announced to subjects and thereafter the process repeats itself. Overall, 420 observations for output and inflation have been generated in the experiments and 4200 individual inflation forecasts have been collected.

Unlike predicted by the rational expectations equilibrium (REE), output and inflation in the baseline experiments show regular and persistent deviations from steady state. In particular, both variables display strong positive autocorrelation. Such behavior emerges because the inflation expectations of subjects participating in the experiments fail to be captured by the expectations functions implied by the REE. This occurs although the REE is determinate, expectationally stable in the sense of Evans and Honkapohja (2001), and has a very simple structure.

The paper shows that instead subjects' inflation expectations are described surprisingly well by a Restricted Perceptions Equilibrium (RPE). The RPE assumes that agents use optimal but 'simple' forecasts, i.e., forecast that minimize mean squared forecast error but that condition on a single explanatory variable only. Interestingly, the REE forecast function itself is simple in this sense because it suggests conditioning inflation forecasts on lagged output. Nevertheless, the restriction to simple forecasts gives rise to an additional RPE in which agents condition inflation forecasts on lagged inflation. Given that agents do so, these forecasts outperform forecasts that condition on lagged output.<sup>3</sup>

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<sup>2</sup>The 'temporary equilibrium' concept is discussed in Grandmont (1988).

<sup>3</sup>Forecasts that condition on lagged inflation do not outperform other forecasts that condition on more than one variable. Such forecasts, therefore, fail to be rational.

The forecast functions implied by the RPE in which agents condition on lagged inflation capture the inflation expectations of subjects in the experiments extremely well. The RPE also predicts output and inflation to display persistent and regular cyclical deviations similar to those observed in the experiments and a sluggish and persistent response of output and inflation to a nominal demand shock.

Over time some divergence of expectations from this RPE can be observed but evidence in favour of convergence to the REE remains weak, even after more than 100 model periods. Yet, for model parameterizations for which the RPE does not exist, subjects' inflation expectations are largely consistent with rational forecast functions.

Overall, the experiments show that deviations of expectations from rational expectations can serve as a powerful endogenous propagation mechanism. Moreover, simple forecast functions seem to describe the forecasts of relatively unexperienced forecasters fairly well and may be a key ingredient for explaining the observed persistence of output and inflation in response to nominal shocks.

Experimental studies of expectation formation go back to Schmalensee (1976) who analyzes price forecasts in a time series context. In this work, however, no feedback from expectations is present, i.e., the time series forecasted by experimental subjects are exogenous and do not depend on their expectations. Subsequent work analyzes price and expectation formation in settings with expectational feedback. A substantial body of research thereby considers experimental asset markets (e.g., Smith et al. (1988); Sunder (1995) for a survey) but little work exists on expectations formation in simple macroeconomic models. A notable exception is Marimon and Sunder (1993), (1994) and Marimon, Spear, and Sunder (1993) who analyze monetary economies with feedback from inflation expectations in a flexible price version of the setting studied in this paper. The focus in these papers is on the relationship between the stability and instability under learning of rational expectations equilibria and the observed laboratory outcomes. The present paper evaluates the relevance of deviations from rational expectations for the persistence of output and inflation.

The remaining part of the paper is structured as follows. Section 2 briefly introduces the economic model and derives the temporary equilibrium equation. The rational expectations and restricted perceptions equilibrium are derived in sections 3 and 4, respectively. Section 5 explains how the experiments have been implemented and sections 6 and 7 discuss the main experimental results. Section 8 studies the robustness of the findings. A conclusion briefly summarizes. Technical details can be found in the appendix.

## 2 A Simple Sticky Price Economy

I consider a simple variant of a monetary sticky price economy with monopolistic competition. This class of models has received considerable attention in monetary economics recently, e.g., Woodford (2003).

The production sector consists of monopolistically competitive firms producing differentiated goods with a linear labor technology. Firms set prices one period in advance but can reoptimize prices every period. Households maximize discounted lifetime utility over consumption and leisure and are subject to a cash-in advance constraint, as in Svensson (1985). Finally, the government generates random demand fluctuations by white noise changes in real money balances. The model has previously been analyzed in Adam (2003, 2005b) and a more detailed description is given in appendix A.1.

Optimal price setting behavior by monopolistic firms delivers a Phillips curve of the form

$$\Pi_t = \frac{1}{1-\sigma} E_{t-1}[\Pi_t w(y_t, E_t \Pi_{t+1})] \quad (1)$$

where  $\Pi_t$  denotes the gross inflation rate,  $y_t$  the output level,  $w(y_t, E_t \Pi_{t+1})$  the marginal cost of production, i.e., the equilibrium real wage, and  $\sigma \in (0, 1)$  the inverse of the elasticity of substitution between the goods of different firms. The real wage  $w$  depends on labor demand, which is equal to output  $y_t$ , and on workers' inflation expectations  $E_t \Pi_{t+1}$ . Inflation expectations influence real marginal costs because of the cash-in-advance constraint faced by workers. Firms must thus forecast inflation two periods ahead even though prices are sticky for a single period only. The real wage  $w(y_t, E_t \Pi_{t+1})$  demanded by workers is assumed to increase with labor demand  $y_t$  and the expected inflation tax  $E_t \Pi_{t+1}$ .<sup>4</sup>

As long as the cash-in-advance constraint is binding, the demand side is given by a quantity equation, which implies that real demand equals real balances. Real money balances and output thus evolve according to

$$y_t = \frac{y_{t-1}}{\Pi_t} + \tau_t \quad (2)$$

where  $\tau_t$  is an *iid* real cash injection of the government with small bounded support and small positive mean  $\tau \geq 0$ .

Linearizing equations (1) and (2) around the monetary steady state delivers an expectational difference equation in output and inflation:<sup>5</sup>

$$\begin{pmatrix} \Pi_t \\ y_t \end{pmatrix} = a_0 + a_1 y_{t-1} + A \begin{pmatrix} {}_{t-1}\Pi_t^e \\ {}_{t-1}\Pi_{t+1}^e \end{pmatrix} + b v_t \quad (3)$$

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<sup>4</sup>Sufficient conditions for this to be the case are given appendix A.1.

<sup>5</sup>See appendix A.1 for details.

where  ${}_{t-1}\Pi_t^e$  and  ${}_{t-1}\Pi_{t+1}^e$  denote the (potentially non-rational)  $t - 1$  expectations of inflation in period  $t$  and  $t + 1$ , respectively, and  $v_t = \tau_t - \tau$  is a mean zero real money shock.<sup>6</sup> The coefficients in equation (3) are given by

$$a_0 = \begin{pmatrix} -\Pi \\ (1 + \frac{1}{\Pi})y \end{pmatrix} \quad a_1 = \begin{pmatrix} \frac{1}{\Pi} \frac{1}{y^\varepsilon} \\ \frac{1}{\Pi} (1 - \frac{1}{\Pi\varepsilon}) \end{pmatrix} \quad (4)$$

$$b = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad A = \begin{pmatrix} 1 - \frac{1}{\Pi\varepsilon} & 1 \\ -\frac{y}{\Pi^2} (1 - \frac{1}{\Pi\varepsilon}) & -\frac{y}{\Pi^2} \end{pmatrix} \quad (5)$$

where  $\Pi \geq 1$  and  $y > 0$  denote steady state inflation and output, respectively, and  $\varepsilon > 0$  is the real wage elasticity of labor supply in the steady state.

Equation (3) describes the models' *temporary equilibrium* in the language of Grandmont (1988), i.e., it determines current output and inflation as a function of predetermined variables and agents' expectations of future endogenous variables. The temporary equilibrium will be used to implement the economy in the experimental laboratory.

### 3 Rational Expectations

Suppose agents' inflation expectations in equation (3) are rational. Then there exists a stationary rational expectations equilibrium (REE) in which

$$y_t = y + v_t \quad (6a)$$

$$\Pi_t = \frac{\Pi}{y} y_{t-1} \quad (6b)$$

The REE displays 'New Classical' properties, i.e., output deviates from steady state only in response to unexpected monetary shocks. Moreover, since prices are predetermined, inflation is lagging output by one period. Overall, the behavior of output and inflation under rational expectations is rather simple and does not depend on the labor supply elasticity  $\varepsilon$ . Moreover, as shown in appendix A.2, no other stationary REE exists.

Clearly, the present model cannot generate persistent deviations of output and inflation from steady state under rational expectations.<sup>7</sup> This inability, which is at odds with estimated impulse responses to monetary shocks, frequently motivates the introduction of additional frictions, e.g., chapter 2 in Woodford (2003).

<sup>6</sup>Conditions under which equation (3) holds even when agents' expectations are not fully rational are given in Adam (2003).

<sup>7</sup>This does not hold for explosive REE, see Adam (2003).

As shown in Adam (2003), the REE (6) is expectationally stable (E-stable) in the sense of Evans and Honkapohja (2001). Least-squares and related learning procedures thus generate convergence to the rational expectations equilibrium. Determinacy, learnability, and simplicity of the REE should facilitate coordination on the equilibrium in the experiments.

## 4 Restricted Perceptions

Instead of assuming rational forecasters, this section considers less sophisticated agents that use 'simple' forecast functions that are based on a single explanatory variable only.<sup>8</sup> In particular, suppose agents consider the following restricted set of forecast models:

$$\text{Model Y : } \quad \Pi_t = \alpha_y + \beta_y y_{t-1} \quad (7a)$$

$$\text{Model II : } \quad \Pi_t = \alpha_{\Pi} + \beta_{\Pi} \Pi_{t-1} \quad (7b)$$

Agents thus forecast inflation either as a function of lagged output, i.e., when employing Model Y, or as a function of lagged inflation, i.e., when employing Model II.<sup>9</sup>

The restriction to *simple forecast functions* can be interpreted in several ways. First, it might reflect the prediction technology available to agents. Given that subjects in the experiments are relatively unexperienced forecasters, this restriction might not be unrealistic. While econometricians in the real world are likely to be more sophisticated than the agents in our experiments, the economic environment in which they operate is also considerably more complicated than suggested by the simple model in this paper. Second, the restriction to simple forecasts may be interpreted as a temporary phenomenon due to agents who perform a specification search for suitable forecast models. Conditioning forecasts either on output or on inflation may appear to be a natural starting point for such forecasters. Unsatisfactory prediction performance, however, may then lead to an enlargement of the set of considered forecast models.

Agents *choose* a simple forecast model from the class  $M \in \{Y, \Pi\}$  but *also* the model parameters  $(\alpha_M, \beta_M)$ . In equilibrium agents choose the model and the parameter values that minimize the mean squared forecast errors associated with their forecast, as is the case in a rational expectations equilibrium.<sup>10</sup> Formally,

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<sup>8</sup>Recent theoretical models involving simple underparameterized forecast functions include Guse (2005) and Branch and Evans (2006).

<sup>9</sup>Of course, agents may use an even simpler forecasting model consisting of an intercept only. The experimental evidence, however, is at variance with such a model.

<sup>10</sup>Squared forecast errors can be interpreted as a quadratic approximation to the correct utility-based choice criterion.

**Definition** A Restricted Perceptions Equilibrium (RPE) in Model  $M^*$  ( $M^* \in \{Y, \Pi\}$ ) is a stationary sequence  $\{y_t, \Pi_t\}_{t=0}^{\infty}$  generated by equation (3) where

1. Agents use Model  $M^*$  with parameters  $(\alpha_M^*, \beta_M^*)$  to forecast future inflation rates where  $(\alpha_M^*, \beta_M^*)$  is the orthogonal projection of  $\Pi_t$  on

$$\begin{cases} (1, y_{t-1}) & \text{if } M^* = Y \\ (1, \Pi_{t-1}) & \text{if } M^* = \Pi \end{cases}$$

2. Model  $M^*$  delivers a lower 1-step ahead mean squared forecast error than model  $M' = \{Y, \Pi\} \setminus \{M^*\}$  where  $(\alpha_{M'}^*, \beta_{M'}^*)$  is the orthogonal projection of  $\Pi_t$  on

$$\begin{cases} (1, y_{t-1}) & \text{if } M' = Y \\ (1, \Pi_{t-1}) & \text{if } M' = \Pi \end{cases}$$

Clearly, without the restriction to the class of simple forecast models (7) a Restricted Perceptions Equilibrium (RPE) is a Rational Expectations Equilibrium (REE): a forecast function that outperforms *all* possible forecast functions is a rational forecast function.

Note that the class of simple forecast models (7) does not rule out that agents hold rational expectations. As shown in section 3, expectations of the simple form (7a) generate rational expectations for  $\alpha_y^* = 0$  and  $\beta_y^* = \frac{\Pi}{y}$ . Thus, the REE is a RPE in Model Y.<sup>11</sup>

To *simplify terminology* the remainder of the paper refers to an RPE in Model Y as the model's Rational Expectations Equilibrium (REE) and reserves the term Restricted Perceptions Equilibrium (RPE) for equilibria where the forecasting constraint is strictly binding. Such a RPE may exist when agents use Model  $\Pi$  for forecasting, i.e.,

$${}_{t-1}\Pi_t^e = \alpha_{\Pi} + \beta_{\Pi}\Pi_{t-1} \quad (8a)$$

$${}_{t-1}\Pi_{t+1}^e = \alpha_{\Pi} + \alpha_{\Pi}\beta_{\Pi} + \beta_{\Pi}^2\Pi_{t-1} \quad (8b)$$

where the 2-step forecast is obtained by iterating on the 1-step forecast function. Substituting these expectations into the temporary equilibrium (3) delivers

$$\begin{pmatrix} \Pi_t \\ y_t \end{pmatrix} = \begin{pmatrix} \alpha_{\Pi}(2 + \beta_{\Pi} - \frac{1}{\Pi\varepsilon}) - \Pi \\ y(1 + \frac{1}{\Pi}) - \frac{y}{\Pi^2}\alpha_{\Pi}(2 + \beta_{\Pi} - \frac{1}{\Pi\varepsilon}) \end{pmatrix} + \begin{pmatrix} \beta_{\Pi}(1 - \frac{1}{\Pi\varepsilon} + \beta_{\Pi}) & \frac{1}{y\varepsilon} \\ -\frac{y}{\Pi^2}\beta_{\Pi}(1 - \frac{1}{\Pi\varepsilon} + \beta_{\Pi}) & \frac{1}{\Pi} - \frac{1}{\Pi^2\varepsilon} \end{pmatrix} \begin{pmatrix} \Pi_{t-1} \\ y_{t-1} \end{pmatrix} + \begin{pmatrix} 0 \\ v_t \end{pmatrix} \quad (9)$$

<sup>11</sup>As is easy to see, no other RPE in Model Y exists.



and shows that inflation then depends on lagged inflation *and* on lagged output. Simple forecast models are thus misspecified and this allows for the possibility that Model II delivers superior predictions than Model Y. Ideally, forecasts, should condition on both variables, lagged inflation and lagged output. Yet, such forecasts fail to be simple and are thus not available to agents.

Adam (2005b) shows that Model II outperforms Model Y whenever the elasticity of labor supply  $\varepsilon$  is larger than some critical value  $\varepsilon^c$ , where  $\varepsilon^c \approx 1.75$  for  $\tau \approx 0$ . For labor supply elasticities below the critical value, the RPE in Model II ceases to exist.<sup>12</sup>

Figure 1 illustrates the response of output and inflation to a nominal demand shock  $v_t$  when the economy is in a RPE in Model Y.<sup>13</sup> Output increases for two periods and inflation reacts in a sluggish but persistent way. Nominal demand shocks thus strongly propagate through the economy unlike in the rational expectations equilibrium.

Figure 2 shows a simulation of the behavior of output and inflation for the RPE in Model II when shocks repeatedly hit the economy.<sup>14</sup> Output and inflation then display persistent deviations from steady state and regular cyclical patterns. Again this contrasts sharply with the rational expectations prediction.

## 5 The Experiments

### 5.1 Implementation

Experiments took place at the University of Salerno, Italy and at the Goethe University of Frankfurt, Germany. In each experimental session five subjects participated with no subject taking part in more than one session. Most subjects were undergraduate business and engineering majors.

At the start of the experiments subjects receive written instructions, which are reproduced in appendix A.5. Instructions inform subjects that they have to repeatedly forecast future inflation rates and how their pay is related to their forecasting performance. Subjects neither know the steady state values of output and inflation nor any other feature of the underlying

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<sup>12</sup>Model Y forecasts then always outperform the predictions generated by Model II and the REE is the only equilibrium.

<sup>13</sup>Figure 1 assumes  $\varepsilon = 2$ ,  $y = 100$ , and  $\Pi = 1.04$ , which is the parameterization used in most of the experiments later on.

<sup>14</sup>Figures 2 assumes  $\varepsilon = 2$ ,  $y = 100$ ,  $\Pi = 1.04$ , and  $v_t \sim iiU[-1, 1]$ , which is the parameterization used in most of the experiments later on.

economy. Subjects are then introduced to the MacroLab software package, which is used to implement the experiments.<sup>15</sup>

The experiment evolves as follows. In any period  $t$  subjects observe the history of output and inflation up to period  $t - 1$  and are asked to forecast inflation for periods  $t$  and  $t + 1$ .<sup>16</sup> Importantly, subjects can enter *any forecast* they wish and subjects know nothing about rational forecasts, Model Y forecasts, or Model II forecasts. Subjects' predictions for period  $t$  and  $t + 1$ , respectively, are then averaged across agents and substituted for the expectations in the temporary equilibrium (3). This together with a draw for the exogenous demand shock  $v_t$  determines output and inflation for period  $t$ .<sup>17</sup> The new values for output and inflation are announced to subjects and subjects receive points for past forecasts of the newly announced inflation rate. Thereafter, the process repeats itself.

The points earned during an experimental treatment are added up and converted into cash payments according to a linear conversion rate announced at the end of the experimental session. Conversion rates are calculated to make an average payment of 30 Euros for subjects participating in single treatment sessions and 60 Euros (50 Euros) for subjects participating in double treatment sessions in Germany (Italy). The average hourly payment per session was never below 12 Euros (8 Euros) in Germany (Italy).

## 5.2 Sessions and Treatments

Six experimental sessions with two different experimental treatments have been implemented. Table 1 lists details of the experimental sessions and treatments.

As a baseline case I consider a **high-elasticity** treatment, in which the elasticity of labor supply is set to  $\varepsilon = 2$ . In high-elasticity treatments a RPE coexists with the REE, whenever agents restrict attention to the set of simple forecasting models (7).

Treatment 1 of Sessions 1 to 4 are the baseline experiments for assessing the performance of the REE and RPE in such high-elasticity treatments. To check for the stability of the results over time, subjects participating in Sessions 3 and 4 are subjected to a second high-elasticity treatment.

As a further robustness check I consider **low-elasticity** treatments in Sessions 5 and 6. The elasticity of labor supply in these sessions is set

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<sup>15</sup>The software is available from the author upon request.

<sup>16</sup>At the start of the experiment subjects observe just a single data point for output and inflation.

<sup>17</sup>Averaging of forecasts is justified on the grounds that it represents a first-order approximation to the exact (non-linear) aggregation of heterogeneous expectations.

to  $\varepsilon = 1$ . The REE is then the only equilibrium, even if agents restrict consideration to simple forecasts functions.

Since the other model parameters do not affect the existence of the REE and RPE, their values are kept constant across all sessions and treatments. In particular, the steady state inflation rate is 4%, the steady state output is 100, and  $v_t \sim iiU[-1, 1]$ .<sup>18</sup>

Treatments in Sessions 1 to 4 last for 55 model periods and treatments in Sessions 5 and 6 for 45 periods.<sup>19</sup> Overall, 420 model periods have been generated and 4200 individual inflation forecasts have been collected.

## 6 Output and Inflation in the Experiments

Figure 3 depicts the typical behavior of output and inflation in the baseline experiments.<sup>20</sup> Both variables display regular and persistent deviations from their steady state values and there seems to be no discernible tendency to behave like white noise processes.

The lower panel of Table 2 reports the autocorrelations of output and inflation for the baseline experiments. The correlations are positive for both variables in all cases and significantly different from zero at the 1% significance level in all but one case. Clearly, such behavior is inconsistent with the REE prediction. It is, however, consistent with the RPE prediction. Output and inflation are then given by

$$y_t \approx 143.699 + .499260 \cdot y_{t-1} - 90.0242 \cdot \Pi_{t-1} + v_t \quad (10a)$$

$$\Pi_t \approx -.472649 + 0.005000 \cdot y_{t-1} + .973701 \cdot \Pi_{t-1} \quad (10b)$$

This follows from equation (9) and

$$\alpha_{\Pi}^* \approx .248842 \quad (11a)$$

$$\beta_{\Pi}^* \approx .760728 \quad (11b)$$

where the equilibrium values  $(\alpha_{\Pi}^*, \beta_{\Pi}^*)$  are derived in appendix A.3. Equations (10) imply

$$corr(y_t, y_{t-1}) \approx 0.70$$

$$corr(\Pi_t, \Pi_{t-1}) \approx 0.76$$

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<sup>18</sup>If follows from appendix A.2 that for this parameterization no other stationary REE exists besides (6).

<sup>19</sup>The length of a treatment is close to 2 hours in some cases, which makes it unwise to choose a higher number of periods.

<sup>20</sup>The working paper version Adam (2005a) provides graphs for all baseline sessions.

which is largely consistent with the autocorrelations of actual output and inflation reported in Table 2. While these findings are suggestive, they are far from conclusive. The next section provides a formal analysis of the experimental data.

## 7 Analysis of Experimental Results

This section analyzes how the REE and RPE perform in explaining the experimental outcomes in the baseline experiments (Sessions 1 to 4, Treatment 1).

Performance of the REE and RPE is evaluated by assessing how well these equilibria explain the inflation forecasts entered by the subjects participating in the experiments. This is motivated by the fact that any deviation of output and inflation from white noise behavior must be driven by deviations of actual expectations from rational expectations.

Throughout the paper consideration is restricted to the inflation forecast of the 'representative subject', i.e., the average forecasts entered by subjects in any given model period. Consideration of such a representative subject is justified on the grounds that output and inflation dynamics in the experimental economies are driven by average forecasts, see section 5.1. Therefore, the equilibrium notion capturing the forecasts of the 'representative agent' also explains the behavior of output and inflation.

Rational inflation forecasts are given by<sup>21</sup>

$${}_{t-1}\Pi_t^{REE} = 0.0104 \cdot y_{t-1} \quad (12a)$$

$${}_{t-1}\Pi_{t+1}^{REE} = 1.04 \quad (12b)$$

while in a RPE inflation expectations are given by<sup>22</sup>

$${}_{t-1}\Pi_t^{RPE} \approx .248842 + .760728 \cdot \Pi_{t-1} \quad (13a)$$

$${}_{t-1}\Pi_{t+1}^{RPE} \approx .438143 + .578708 \cdot \Pi_{t-1} \quad (13b)$$

The subsequent sections compare the REE prediction (12) and the RPE prediction (13) with the actual inflation forecasts entered by subjects in the experiments.

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<sup>21</sup>This follows from equations (6) and the parameterization described in section 5.2.

<sup>22</sup>This follows from equations (8) and (11).

## 7.1 Unconditional Inflation Forecasts

In a first step I consider the average inflation forecasts across agents *and* model periods. Such average forecasts can be interpreted as an estimate of the representative agent's unconditional inflation forecast.

REE and RPE both predict unconditional 1-step and 2-step forecasts to be equal to the steady state inflation rate. Unconditional forecasts, therefore, do not allow to discriminate between the REE and RPE. Yet, since agents do not know the steady state inflation rate, unconditional forecasts allow to evaluate whether REE and RPE capture the first moment of subjects' inflation expectations. A failure to do so would be a failure of first order importance.

Table 3 lists the average 1-step and 2-step inflation forecasts for all experimental sessions. Of the 16 values reported, 12 are within 2 standard deviations and 15 within 3 standard deviations of the predicted steady state value of 4%.<sup>23</sup> The average inflation forecasts thus are consistent with the predictions of the REE and RPE.

The subsequent sections will consider conditional inflation forecasts. Conditional forecasts differ across the REE and RPE and thus allow to discriminate between the two equilibria.

## 7.2 Conditional Inflation Forecasts

Figures 4 and 5 depict examples of subject's actual inflation forecasts together with the REE prediction (12) of these forecasts.<sup>24</sup> It is obvious from these figures that REE forecasts perform rather poorly in explaining agents' actual inflation forecasts. Agents' 1-step forecasts clearly lag the REE prediction and agents' 2-step forecasts display regular cyclical patterns that is at odds with the REE prediction of a constant forecast. Also, there seems to be no tendency of actual forecasts to converge over time to the behavior predicted by REE forecasts.

Figures 6 and 7 depict subject's actual inflation forecasts together with the RPE prediction (13) of the forecasts for the same sessions as before. Agents' 1-step forecasts are captured surprisingly well by the RPE-forecasts. The same holds for agents' 2-step forecasts, except for Session 3 where the performance of RPE forecasts seems to deteriorate somewhat towards the end of the treatment.

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<sup>23</sup>The low values in Session 6 are partly driven by one subject which entered large negative forecasts in period 12 of the experiment.

<sup>24</sup>Due to space constraints the Figures report result for Sessions 3 and 4 only. Figures for Sessions 1 and 2 are almost identical and are available in the working paper version Adam (2005a).

The evidence in Figures 4 to 7 provides considerable support in favor of the RPE. Overall, the close fit between RPE-forecasts and actual forecasts is remarkable since RPE-forecasts are calculated *without* taking into account agents' actual forecasts: RPE forecasts rely only on information that is available to agents at the time of forecasting.<sup>25</sup>

The visual impression from Sessions 3 and 4 can be confirmed by a more formal analysis (applied to all baseline experiments). To assess how well the REE and RPE explain the inflation forecasts of the experiments, I report OLS-estimates of the parameter  $\beta$  for the following regression

$${}_{t-1}\Pi_{t+i}^{actual} = \alpha + \beta \cdot {}_{t-1}\Pi_{t+i}^{RPE} + (1 - \beta) \cdot {}_{t-1}\Pi_{t+i}^{REE}$$

where  ${}_{t-1}\Pi_{t+i}^{actual}$  denotes the actual time  $t - 1$  forecast of the  $t + i$  inflation rate ( $i = 0, 1$ ) and  ${}_{t-1}\Pi_{t+i}^{REE}$  and  ${}_{t-1}\Pi_{t+i}^{RPE}$  the corresponding equilibrium forecasts in a REE and RPE, respectively, as given by equations (12) and (13). The estimate for  $\beta$  can be interpreted as the share of agents using the RPE-forecasts. A value of  $\beta$  close to 1 indicates that the RPE explains the forecast functions well, while a value close to zero indicates that the REE offers a superior description of the forecast function.

The first panel of Table 4 reports the estimated share of RPE-forecasters  $\beta$  for the 1-step inflation forecasts. Estimates are reported for the entire treatment and the last 20 periods to assess whether there is some variation over time due to learning processes taking place.

The point estimates in the upper panel of Table 4 are relatively close to 1 and imply that in each treatment more than 85% of agents use RPE-forecasts. The shares are estimated rather precisely and there are only weak signs (in Session 1 and 3) that they are significantly lower in the last 20 periods of the treatments.

The second panel of Table 4 reports the  $\beta$  estimates for the 2-step forecasts. The RPE-forecasts clearly dominate in Sessions 2 and 4. Also, in these sessions the dominance of RPE-forecasts appears to be stable over time. RPE-forecasts also perform well in the first part of Sessions 1 and 3 but towards the end of the treatment the performance of RPE deteriorates.<sup>26</sup> This should hardly be surprising given the evidence shown in the lower panel of Figure 6.

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<sup>25</sup>A surprising feature of the experimental data is the speed at which agents seem to be able to coordinate on the RPE. I have not been able to come up with a convincing explanation for this phenomenon.

<sup>26</sup>Although the point estimate for  $\beta$  is low for the last 20 periods of Sessions 1 and 3, REE forecasts fail to capture actual 2-step forecasts, as should be clear from Figure 4. In the regressions this manifests itself by low  $R^2$  values.

Overall, however, the baseline experiments provide overwhelming evidence in favor of the RPE. While the RPE captures agents inflation forecasts in most of the sessions, the REE does not offer a good description of subjects' inflation expectations in any of the baseline sessions.

Figure 8 provides additional support for this claim by depicting actual inflation rates together with the forecasted rates.<sup>27</sup> For rational forecasts the difference between the actual and forecasted inflation series is white noise. However, as is easy to spot in Figure 8, for both forecast horizons inflation forecasts lag actual inflation.<sup>28</sup> As a result, forecast errors are strongly positively autocorrelated, a feature consistent with the RPE but not with the REE.

The next subsection analyzes the data from Sessions 1 and 3 in greater detail. In these sessions the performance of RPE 2-step forecasts deteriorated towards the end of the treatment.

### 7.3 What Happened in Sessions 1 and 3?

Figure 9 depicts actual 2-step forecasts from Sessions 1 and 3 together with the following output-based forecast function:

$${}_{t-1}\Pi_{t+1}^e \approx 0.4172 + 0.0062y_{t-1} \quad (14)$$

Towards the end of the treatments agents' 2-step forecasts seem to be captured rather well by equation (14). Moreover, forecasts (14) start to perform well precisely when the performance of the RPE-forecasts starts to deteriorate, see the lower panel of Figure 6. This suggests that agents participating in Sessions 1 and 3 have substituted their RPE 2-step forecast function (13b) with the output-based forecast function (14).

Forecast function (14) is the optimal simple 2-step inflation forecast for an economy that is in a RPE, as the data suggest to be the case for the first half of the considered sessions.<sup>29</sup>

It might come as a surprise that the optimal simple 2-step forecast function differs from the RPE 2-step forecast function (13b). RPE 2-step forecasts are suboptimal because they are obtained by iterating forward the optimal simple 1-step forecast equation (13a). Although iteration is the standard procedure in econometrics to derive a multi-step prediction from

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<sup>27</sup>The figure uses data from Session 4, Treatment 1. The evidence for the other sessions looks very similar.

<sup>28</sup>The figure depicts the actual inflation rate and past forecasts of this inflation rate at the same point of the  $x$ -axis, so this feature is not due to a problem of representation.

<sup>29</sup>Optimality should again be interpreted as minimizing the mean squared forecast errors.

a linear econometric model, it is suboptimal here since the 1-step forecast function is misspecified, e.g., Bhansali (2002) for a discussion.

Instead of iterating 1-step forecasts, a superior simple 2-step forecast is obtained by regressing inflation directly on twice lagged output or twice lagged inflation. Doing so one finds that function (14) delivers the optimal simple forecast when the economy is in a RPE.

The somewhat informal discussion above can be supported by a more formal analysis. The lower panel of Table 4 lists the OLS-estimate of  $\beta$  obtained from the regression

$${}_{t-1}\Pi_{t+1}^{actual} = \alpha + \beta \cdot {}_{t-1}\Pi_{t+1}^{RPE} + (1 - \beta) \cdot {}_{t-1}\Pi_{t+1}^{Output}$$

where  ${}_{t-1}\Pi_{t+1}^{Output}$  denotes the forecast given in equation (14).

While Sessions 2 and 4 do not show any signs of subjects switching towards the output-based forecast rule (14), there are strong indications for such behavior in Sessions 1 and 3. The point estimates imply that the large majority of agents used output-based 2-step forecasts during the last 20 periods of the treatment.

The previous findings suggest that agents' in Sessions 1 and 3 have become aware that their iterated 2-step forecast is suboptimal and have started to use different forecast models for different forecast horizons.

## 8 Robustness

This section studies the robustness of the previous results.

### 8.1 Additional High-Elasticity Treatments

To analyze the stability of the RPE over time, subjects participating in Sessions 3 and 4 have been subjected to an additional high-elasticity treatment.

Subjects in some of the baseline experiments seemed to have switched to output-based 2-step forecasts towards the end of the treatment. As shown in appendix A.4, there exists a unique stationary rest point where agents use an optimal inflation-based 1-step forecast and an optimal output-based 2-step forecast.<sup>30</sup> This rest point will be referred to as the *mixed-forecast situation* subsequently and could be expected to emerge in the additional

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<sup>30</sup>Optimality is again defined in terms of mean-squared forecast errors.



high-elasticity treatments analyzed in this section. The forecast functions in the mixed-forecast situation are given by

$${}_{t-1}\Pi_t^e \approx 0.6887 + 0.3378 \cdot \Pi_{t-1} \quad (15a)$$

$${}_{t-1}\Pi_{t+1}^e \approx 0.7373 + 0.003027 \cdot y_{t-1} \quad (15b)$$

Importantly, these equations do *not* describe an equilibrium, i.e., a situation where agents use optimal simple forecast functions for *each* forecast horizon. If agents use equations (15), a 1-step forecast with output as the explanatory variable would dominate the inflation-based forecast (15a). Clearly, once agents substitute the inflation-based 1-step forecast (15a) by an output-based forecast, the REE should emerge. To assess whether convergence to the REE occurs, this section thus compares the mixed-forecasts (15) against the REE forecasts (12).<sup>31</sup>

Figures 10 and 11 depict actual forecasts, REE forecasts, and mixed forecasts. Interpretation of the data from Session 3 is somewhat difficult because one subject experimented with large negative inflation forecasts in period 16-22 to learn about the economy's reaction to these forecasts.<sup>32</sup>

The figures suggest that in both sessions 1-step forecasts are captured far better by the (inflation-based) mixed-forecast than by the (output-based) REE-forecast. This is confirmed by the quantitative evidence presented in the upper panel of Table 5. The share of agents using the (inflation-based) mixed forecast function is estimated to be close to one and is not significantly lower in the last 20 periods of the treatments. Thus, regarding 1-step forecasts there is no evidence in favor of a convergence process towards the REE.<sup>33</sup>

Figures 10 and 11 equally suggests that 2-step forecasts seem to be more in line with the mixed-forecasts than with the REE-forecasts, in particular towards the end of the treatments. The visual impression is again confirmed by the quantitative results reported in the second and third panel of Table 5. The second panel shows that the mixed-forecast function dominates in Session 4 and seems to gain weight in Session 3, where interpretation is hampered by the fact that one subject experimented with large negative

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<sup>31</sup>Importantly, the learning process from the RPE towards the mixed-forecast situation should not cause major problems for the analysis: the coefficient on lagged inflation in equation (15a) has the same sign as in the RPE and the coefficient on lagged output in equation (15b) has the same sign as in equation (14).

<sup>32</sup>The subject mentioned to me that he experimented after the end of the experiment. He also mentioned that he had abandoned experimentation after a while as it became too costly and did not generate a lot of information.

<sup>33</sup>There is some evidence that actual forecasts initially fluctuate more than the mixed 1-step forecasts (15a) suggest. This is likely to be the case because agents still use the RPE-forecast, which has a larger coefficient on the lagged inflation term.

forecasts. The third panel illustrates that almost all agents seem to use output-based forecast rules.

Overall, the additional high-elasticity treatments suggest that agents' forecast functions moved into the direction of the mixed forecast situation (15), which offers a better description of subjects' inflation forecasts than both the REE forecasts and the RPE forecasts. Thus, after more than 110 model periods the REE does still not emerge as the dominant explanation of the data.

## 8.2 Low Elasticity Treatments

This section discusses the results from the low-elasticity treatments in Sessions 5 and 6. In a low-elasticity economy the RPE does not exist even for agents restricting attention to the class of simple forecast functions (7).

Figure 12 depicts actual forecasts and REE-forecasts from Session 5. Towards the end of the experimental treatment, REE and actual 1-step forecasts move in a highly synchronized fashion and the fit seems to be much better than after the two high-elasticity treatments applied in Sessions 3 and 4. The 2-step forecasts still display some cyclical variation, but the standard deviation of 2-step forecasts is rather small.

Figure 13 displays information on Session 6. Since one subject entered large negative values for the inflation forecasts in period 12, the signs in favor of convergence towards the REE are much weaker for this session.<sup>34</sup>

The visual impression can again be confirmed by a more formal analysis. Since an RPE does not exist in low-elasticity treatments, one has to compare the ability of REE-forecasts to explain actual forecasts with the ability to do so in high elasticity treatments. Table 6 reports results of such a comparison for the 1-step forecasts. The table reports for all sessions and treatments the coefficient  $\delta$  obtained from estimating equation

$${}_{t-1}\Pi_t^{actual} = \gamma + \delta \cdot {}_{t-1}\Pi_t^{REE} \quad (16)$$

using ordinary least squares.

While for the low-elasticity treatments  $\delta$  is significant and positive, it is either insignificant or negative for the high-elasticity treatments. This together with the fact that  $\delta$  is very high in the last 20 periods of Session 5 suggests that in low-elasticity treatments the REE-forecasts offers a

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<sup>34</sup>It is not entirely clear whether the forecast was an attempt to obtain information about the economy's reaction to such forecasts or whether this was a simple omission of the decimal point.

much better explanation of actual 1-step forecasts than in high-elasticity treatments.

Table 7 reports evidence on 2-step forecasts. The table presents results from regressing on a constant the squared deviation of actual 2-step forecasts from REE 2-step forecasts. For Session 5 the estimated constant is significantly lower than in all other high elasticity sessions, both for the whole sample and for the last 20 periods. For Session 6 the picture is more mixed. Nevertheless, for the last 20 periods of Session 6 the squared deviation is still significantly lower than in all but one high-elasticity treatment.

Summing up, the previous evidence indicates that in low-elasticity treatments the REE performs significantly better in explaining agents' inflation expectations than in high-elasticity sessions.

## 9 Conclusions

This paper has shown that deviations of expectations from rational expectations can play an important role in enhancing the internal propagation mechanism of simple macroeconomic models. Less than rational expectations can generate considerable persistence of output and inflation in response to nominal shocks where rational expectations predict these variables to display no persistence at all. Non-rationalities in agents' expectations may thus provide a substitute for a range of other frictions generally required to generate such behavior under rational expectations.

Importantly, the presence of deviations from forecast rationality may be systematically linked to underlying characteristics of the economy. Consistent with the assumed theory of expectations formation, it has been found that the RPE explains the experimental outcome in high-elasticity economies while the REE tends to explain the outcomes in low-elasticity economies.

Overall, the experimental outcomes suggest that the forecasting technology employed by relatively unsophisticated forecasters can be captured by simple forecast functions that condition on a single explanatory variable. It seems important to learn whether such simple forecast functions also capture subjects' expectations in other experimental economies or in field data.

## A Appendix

### A.1 Details of the structural model

Below I outline a highly stylized business cycle model with monopolistic competition (Dixit and Stiglitz (1977)) where firms set prices one period in advance. In slight deviation from a standard setup there are two kinds of agents: a unit mass of entrepreneurs who own monopolistically competitive firms and finance consumption using the monopolistic profits; a unit mass of workers who finance consumption by offering their work force on a competitive labor market. To make the distinction between workers and entrepreneurs economically relevant, contingent claim markets that would allow for risk sharing between workers and entrepreneurs are assumed to be unavailable. The previous setting insures that workers' labor supply decisions do not depend on current and expected future profits and this simplifies the analysis.

Entrepreneur  $i \in [0, 1]$  maximizes a standard utility function of the form

$$\begin{aligned} \max_{\{c_t^i\}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^i) \quad s.t. \\ c_t^i \leq \frac{m_{t-1}^i}{\Pi_t} + \frac{1}{2} \tau_t \\ m_t^i = \frac{m_{t-1}^i}{\Pi_t} - c_t^i + \Phi_t^i + \frac{1}{2} \tau_t \end{aligned}$$

where  $c_t^i$  denotes consumption,  $m_t^i$  the entrepreneur's real money holdings at the end of period  $t$ ,  $\tau_t$  the real value of a possibly negative government cash transfer,  $\Pi_t$  the inflation factor from period  $t-1$  to  $t$ , and  $\Phi_t^i$  the monopoly rents from ownership of firm  $i$ . The first constraint forces entrepreneurs to use money to pay for consumption goods. The second constraint is the flow budget constraint.

Each firm  $i$  produces an intermediate consumption good  $q^i$  which is an imperfect substitute in the production of the aggregate consumption good  $c$ :

$$c = \left( \int_{i \in [0,1]} (q^i)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \quad \text{with } 1 > \sigma \geq 0$$

Profit maximization implies that each firm sets its price  $P_t^i$  as a fixed markup over expected production costs. With prices being fixed one period in advance and the production technology being linear in labor this implies

$$P_t^i = \frac{1}{1-\sigma} E_{t-1}[P_t w_t]$$

where  $P_t$  denotes the price index of the final consumption good and  $w_t$  the real wage. Dividing the previous equation by  $P_{t-1}$  and assuming that all entrepreneurs and firms have identical expectations delivers

$$\Pi_t = \frac{1}{1-\sigma} E_{t-1}[\Pi_t w_t] \quad (17)$$

Equation (17) summarizes optimal behavior by firms.

Next, consider workers. Each worker  $j \in [0, 1]$  chooses consumption  $c_t^j$  and labor supply  $n_t^j$  to maximize

$$\begin{aligned} \max_{\{c_t^j, n_t^j\}} E_0 \sum_{t=0}^{\infty} \beta^t \left( u(c_t^j) - v(n_t^j) \right) \quad s.t. \\ c_t^j \leq \frac{m_{t-1}^j}{\Pi_t} + \frac{1}{2} \tau_t \\ m_t^j = \frac{m_{t-1}^j}{\Pi_t} - c_t^j + n_t^j w_t + \frac{1}{2} \tau_t \end{aligned}$$

When  $u, v \in C^2$ ,  $u' > 0$ ,  $u'' < 0$ ,  $v' > 0$ ,  $v'' \geq 0$ ,  $-\frac{u''(c) \cdot c}{u'(c)} < 1$  for all  $c \geq 0$ , and the cash-in-advance constraint is binding, utility maximization implies a labor supply function of the form:<sup>35</sup>

$$n_t = n(w_t, E_t[\Pi_{t+1}])$$

Inverting this labor supply function with respect to the first argument delivers an expression for the real wage:

$$w_t = w(y_t, E_t[\Pi_{t+1}]) \quad \text{with} \quad \frac{\partial w}{\partial y} > 0, \frac{\partial w}{\partial E_t[\Pi_{t+1}]} > 0 \quad (18)$$

where the linearity of the production function has been used to substitute  $n_t$  by  $y_t$ . Given the specified utility functions, the real wage increases in the demand for labor and in the expected inflation tax.

Finally, consider the government which issues money via lump sum transfers.<sup>36</sup> The government's behavior implies that real money balances evolve according to

$$m_t = \frac{m_{t-1}}{\Pi_t} + \tau_t$$

where  $\tau_t$  is a mean zero white noise shock with small bounded support and is the only source of randomness in the model. When prices are preset and

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<sup>35</sup>For simplicity I assume that agents hold point expectations, as is standard in the learning literature. Once the model is linearized, however, the point expectations may be interpreted as the mean of expectations with non-trivial support.

<sup>36</sup>Alternatively, the government may issue money to purchase goods for government consumption.

the cash-in-advance constraint is binding for all agents, output is demand determined in the short run and the previous equation is a specification of the demand side of the economy.<sup>37</sup> This implies that output can be written as

$$y_t = \frac{y_{t-1}}{\Pi_t} + \tau_t \quad (19)$$

which is equation (2). Substituting (18) into (17) delivers (1).

The linearization coefficients (4) and (5) are obtained from (17) and (18) using the following equations to simplify expressions:

$$w = 1 - \sigma \quad (20)$$

$$y = \frac{y}{\Pi} + \tau \quad (21)$$

$$\frac{\partial w(y, \Pi')}{\partial \Pi'} = \frac{1 - \sigma}{\Pi} \quad (22)$$

Equations (20) and (21) are steady state versions of (17) and (18), respectively, and (22) follows from applying the implicit function theorem to workers' first order condition with a binding cash-in-advance constraint.

## A.2 Uniqueness of REE

To prove uniqueness, I derive the Blanchard-Kahn form of model (3). Since  $\Pi_t$  is predetermined,  $E_t \Pi_{t+1} = \Pi_{t+1}$  in any REE. This allows us to write model (3) as

$$\begin{pmatrix} 1 & 0 & 0 \\ -\frac{1}{y\varepsilon} & \frac{1}{\Pi\varepsilon} & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} y_t \\ \Pi_{t+1} \\ E_t \Pi_{t+2} \end{pmatrix} = \begin{pmatrix} \frac{1}{\Pi} & -\frac{y}{\Pi^2} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} y_{t-1} \\ \Pi_t \\ \Pi_{t+1} \end{pmatrix} + \begin{pmatrix} y \\ -\Pi \\ 0 \end{pmatrix} + \begin{pmatrix} v_t \\ 0 \\ 0 \end{pmatrix}$$

where  $y_{t-1}$  and  $\Pi_t$  are 'predetermined'. Inverting the matrix on the l.h.s., the resulting matrix in front of the endogenous variables on the r.h.s. has two eigenvalues equal to zero and another eigenvalue equal to  $\frac{\varepsilon+1}{\Pi\varepsilon}$ . Thus, as long as  $\frac{\varepsilon+1}{\varepsilon\Pi} > 1$ , there exists a unique stationary REE. For  $\tau$  sufficiently small, as has been assumed,  $\Pi \approx 1$  at the low inflation steady state and this inequality holds as long as  $\varepsilon > 0$ .

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<sup>37</sup> Along a deterministic equilibrium path where the expected inflation factor is above the discount factor, the cash-in-advance constraint strictly binds if initial money balances are not too high. Also, in the stochastic case with small support for the shocks, surprise deflation and shocks to real cash holdings will be small implying that agents will always wish to spend their entire money balances for consumption.

### A.3 Calculating $(\alpha_{\Pi}, \beta_{\Pi})$ in Model $\Pi$ Equilibrium

In a stationary equilibrium the least squares estimates of Model  $\Pi$  are given by

$$\beta_{\Pi} = \frac{\text{cov}(\Pi_t, \Pi_{t-1})}{\text{var}(\Pi_{t-1})} \quad (23)$$

$$\alpha_{\Pi} = \Pi(1 - \beta_{\Pi}) \quad (24)$$

where  $\Pi$  is the steady state inflation rate and where  $\Pi_t$  evolves according to (9). Let  $B$  denote the AR-matrix in (9) and  $\text{vec}$  be the column-wise vectorization operator. Then taking variances on both sides of (9) and assuming stationarity implies that

$$\text{vec}(\Sigma) = (I - B \otimes B)^{-1} \text{vec}(\Omega) \quad (25)$$

where  $\Sigma$  is the covariance matrix for  $(\Pi_t, y_t)$  and

$$\Omega = \begin{pmatrix} 0 & 0 \\ 0 & \sigma_v^2 \end{pmatrix}$$

Equation (25) delivers the expression for the denominator in (23). From

$$\Gamma = \begin{pmatrix} \text{cov}(\Pi_t, \Pi_{t-1}) & \text{cov}(\Pi_t, y_{t-1}) \\ \text{cov}(y_t, \Pi_{t-1}) & \text{cov}(y_t, y_{t-1}) \end{pmatrix} = B\Sigma$$

one obtains the expression for the numerator in (23). Equation (23) thus implies that  $\beta_{\Pi}$  solves

$$\beta_{\Pi} = \frac{\beta_{\Pi}(1 - \frac{1}{\Pi\varepsilon} + \beta_{\Pi}) + \frac{1}{\Pi} - \frac{1}{\Pi^2\varepsilon}}{1 + \beta_{\Pi}(1 - \frac{1}{\Pi\varepsilon} + \beta_{\Pi}) (\frac{1}{\Pi} - \frac{1}{\Pi^2\varepsilon}) + \frac{1}{\Pi^2}\beta_{\Pi}(1 - \frac{1}{\Pi\varepsilon} + \beta_{\Pi})\frac{1}{\varepsilon}} \quad (26)$$

For the high-elasticity parameterization this equation has one real and two imaginary solutions for  $\beta_{\Pi}$ . The value for  $\alpha_{\Pi}$  can be obtained from equation (24).

### A.4 The Mixed Forecast Situation

Suppose agents use the following 1-step and 2-step forecast model:

$$\Pi_t = \alpha + \beta\Pi_{t-1} \quad (27a)$$

$$\Pi_{t+1} = \gamma + \delta y_{t-1} \quad (27b)$$

where in a stationary equilibrium

$$\beta = \frac{\text{cov}(\Pi_t, \Pi_{t-1})}{\text{var}(\Pi_{t-1})} \quad (28)$$

$$\delta = \frac{\text{cov}(\Pi_t, y_{t-2})}{\text{var}(y_{t-2})} \quad (29)$$

and

$$\begin{aligned}\alpha &= (1 - \beta)\Pi \\ \gamma &= \Pi - \delta y\end{aligned}$$

where variables without subscript denote steady state values.

To calculate the variances and covariances insert the expectations derived from (27) into (3). This delivers

$$\begin{pmatrix} \Pi_t \\ y_t \end{pmatrix} = a + B \begin{pmatrix} \Pi_{t-1} \\ y_{t-1} \end{pmatrix} + \begin{pmatrix} 0 \\ v_t \end{pmatrix} \quad (30)$$

where

$$B = \begin{pmatrix} (1 - \frac{1}{\Pi\varepsilon})\beta & \delta + \frac{1}{\Pi\varepsilon} \\ -\frac{y}{\Pi^2}(1 - \frac{1}{\Pi\varepsilon})\beta & -\frac{y}{\Pi^2}\delta + (\frac{1}{\Pi} - \frac{1}{\Pi^2\varepsilon}) \end{pmatrix}$$

Since the constant  $a$  in (30) does not influence the covariances one can ignore it from now on. Taking variances on both sides of (30) delivers

$$vec(\Sigma) = vec\left(VAR\left(\begin{pmatrix} \Pi_t \\ y_t \end{pmatrix}\right)\right) = (I - B \otimes B)^{-1}vec(\Omega)$$

where  $vec(\cdot)$ ,  $\Sigma$ , and  $\Omega$  are as defined in appendix A.3. Multiplying (30) by the once and twice lagged  $(y_t, \Pi_t)$  row-vector and taking expectations delivers

$$\begin{aligned}\Gamma_1 &= B\Sigma \\ \Gamma_2 &= B^2\Gamma_1\end{aligned}$$

where  $\Gamma_i$  is the covariance matrix of output and inflation with  $i$ -times lagged output and inflation. Taking the variances and covariances from these expressions delivers

$$\beta = \frac{cov(\Pi_t, \Pi_{t-1})}{var(\Pi_{t-1})} = \frac{-1 - \beta\Pi(1 - \Pi\varepsilon) + \varepsilon(\Pi - \delta y)}{\varepsilon\Pi^2 - \beta(1 - \Pi\varepsilon)}$$

This equation for  $\beta$  has two fixed points given by

$$\begin{aligned}\beta_1 &= \frac{-\Pi - \sqrt{\Pi^2 - 4(-1 + \varepsilon\Pi)(1 - \varepsilon\Pi + \delta\varepsilon y)}}{2(-1 + \varepsilon\Pi)} \\ \beta_2 &= \frac{-\Pi + \sqrt{\Pi^2 - 4(-1 + \varepsilon\Pi)(1 - \varepsilon\Pi + \delta\varepsilon y)}}{2(-1 + \varepsilon\Pi)}\end{aligned}$$

which depend on  $\delta$ . Substituting  $\beta_1$  into (29) using the expressions for the covariances derived above and solving for  $\delta$  with  $\Pi = 1.04$  and  $\varepsilon = 2$  delivers two real and two imaginary solutions for  $\delta$ . However, both real solutions imply values for  $\beta_1$  smaller than  $-1$  which would contradict stationarity of the inflation rate.

Substituting  $\beta_2$  into (29) and solving for  $\delta$  with  $\Pi = 1.04$  and  $\varepsilon = 2$  delivers three real solutions for  $\delta$ . Only one of these solutions implies a value of  $|\beta_2| < 1$ . This is the solution shown in equation (15).



## **A.5 Instructions for Subjects**

### **General**

Today you will participate in an experiment of economic decision making. Various research foundations have provided funds for the conduct of this research. Instructions are simple and if you follow them carefully you can earn a considerable amount of money. The average payment will be around 60.000 Lire but, depending on how well you do, you may well earn up to 120.000 Lire.

You are assigned the role of a private agent whose task is to forecast the rate of inflation in the economy. In each experimental period  $t$ , you are asked to forecast the inflation rate for the next two periods, i.e. the inflation rate for period  $t+1$  and for period  $t+2$ .

In period  $t$  when you make your inflation predictions for  $t+1$  and  $t+2$  you can observe the current and past data of the economy. This data consists of the current and past inflation rates and the current and past levels of real GDP, where real GDP is the quantity of goods that is produced in the economy.

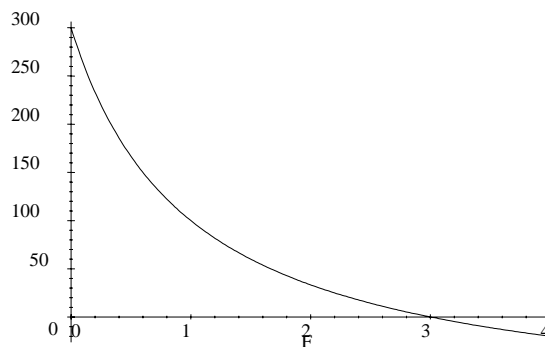
At the beginning of an experiment when you start forecasting, there is just a single data point that consists of the current inflation rate and output level. After you have made your forecasts the experiment period will end and a new experiment period will start for which a new inflation rate and output level will be announced. Thus, as the experiment evolves you will have an increasing number of observations.

There will be various experiment 'sessions'. For each 'session' the economy will restart from period zero. Each session is unrelated to the previous session, in the sense that the level of inflation and output will be different across sessions. Also the relationships between inflation and output and past values of these variables is not necessarily the same from one session to another. The end of a session and the beginning of a new session will be clearly announced by the experimenter.

### **Earnings**

During each period of an experiment session you will collect 'points' which at the end of the session will be transformed into Lira, as described below. The number of points that you get will depend on how close your inflation predictions are to the actual inflation rates. The details are explained now:

Each period  $t$ , the new inflation rate and the new output level are announced. You will have predicted the current inflation rate two different times, once 1 period ago and once 2 periods ago.



Let  $f$  denote the absolute value of your forecast error from one of these forecasts. The error is expressed in percentage points, i.e. if  $f = 1.5$  your forecast was either 1.5% higher or lower than the actual inflation rate.

The points that you receive will depend on the errors  $f$  you make where larger errors will give you less points. In particular, points are calculated in the following way

$$\frac{400}{1 + f} - 100$$

You can receive up to 300 points per forecast and may lose up to 100 points depending on the size of the forecast error. With a zero forecast error you would receive 300 points. However, if your forecast is 1% higher or lower than the actual inflation rate you will get only 100 points ( $400/2 - 100$ ), likewise for a 3% forecast error you receive no points ( $300/3 - 100$ ), and for even larger forecast errors points will be subtracted. The graph below shows the relationship between the forecast error and the points that you receive for your forecast.

You will receive points for each of the two forecast you made for the current inflation rate, i.e. you receive points for the forecast you made 1 period ago and points for the forecast you made 2 periods ago, where the number of points for each forecast depends on the forecast error as described above.

After the experimenter has announced the end of an experiment session, write down the total number of points that you received on a sheet of paper with your name on it. Briefly after the end of the session, the experimenter will announce a conversion rate that indicates the value of the points in terms of Lira.

### Other Instructions

During the experiment sessions it is strictly forbidden to speak with other students that participate in the experiment. Doing so can lead to the exclusion from the experiment. In this case no payment will be made. If you

have any questions or problems during the course of the experiment raise your hand and the experimenter will come to you.

At the start of each experiment session you will be asked to start the program that runs on your computer. Please carefully follow the instructions that you will receive from the experimenter.

**If you have any questions please ask them now!**

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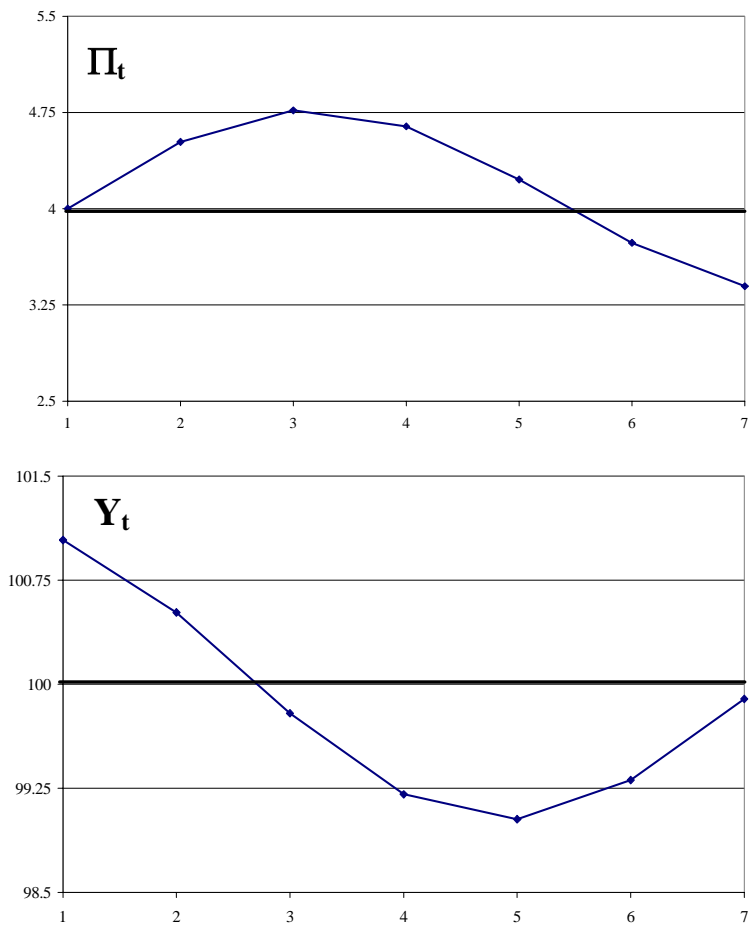


Figure 1: Impulse Response to a Nominal Disturbance in the Restricted Perceptions Equilibrium

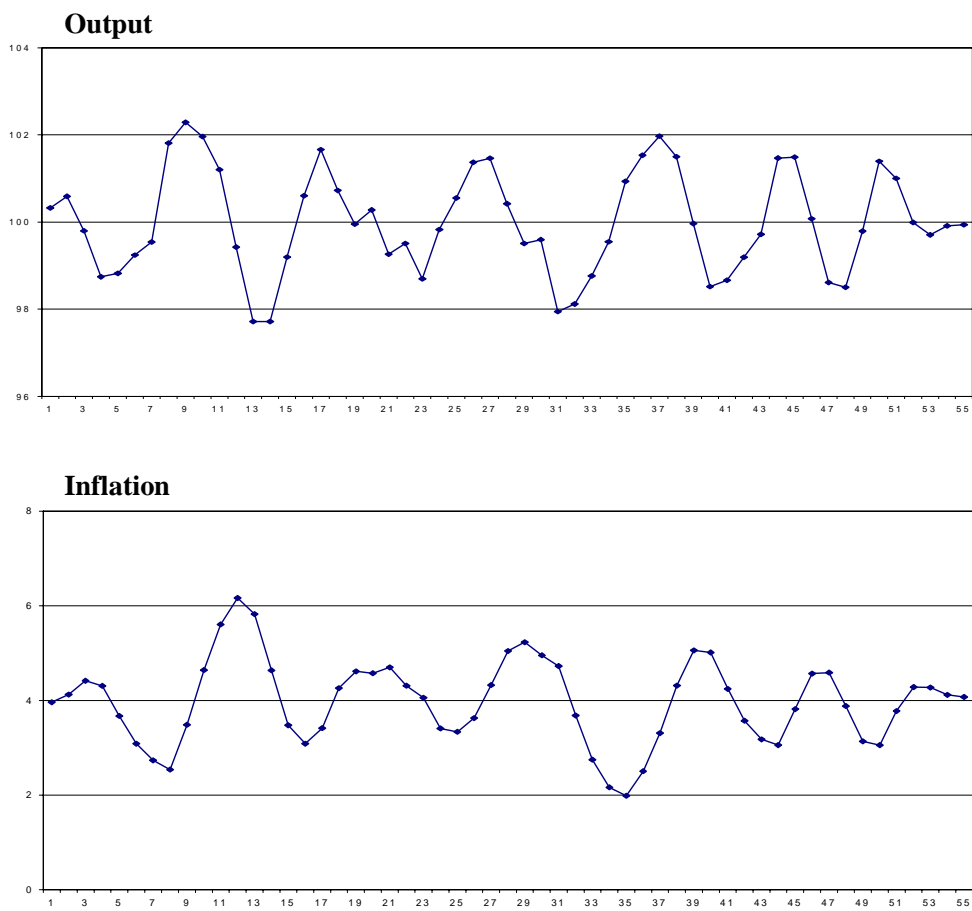


Figure 2: Output and Inflation in the Restricted Perceptions Equilibrium

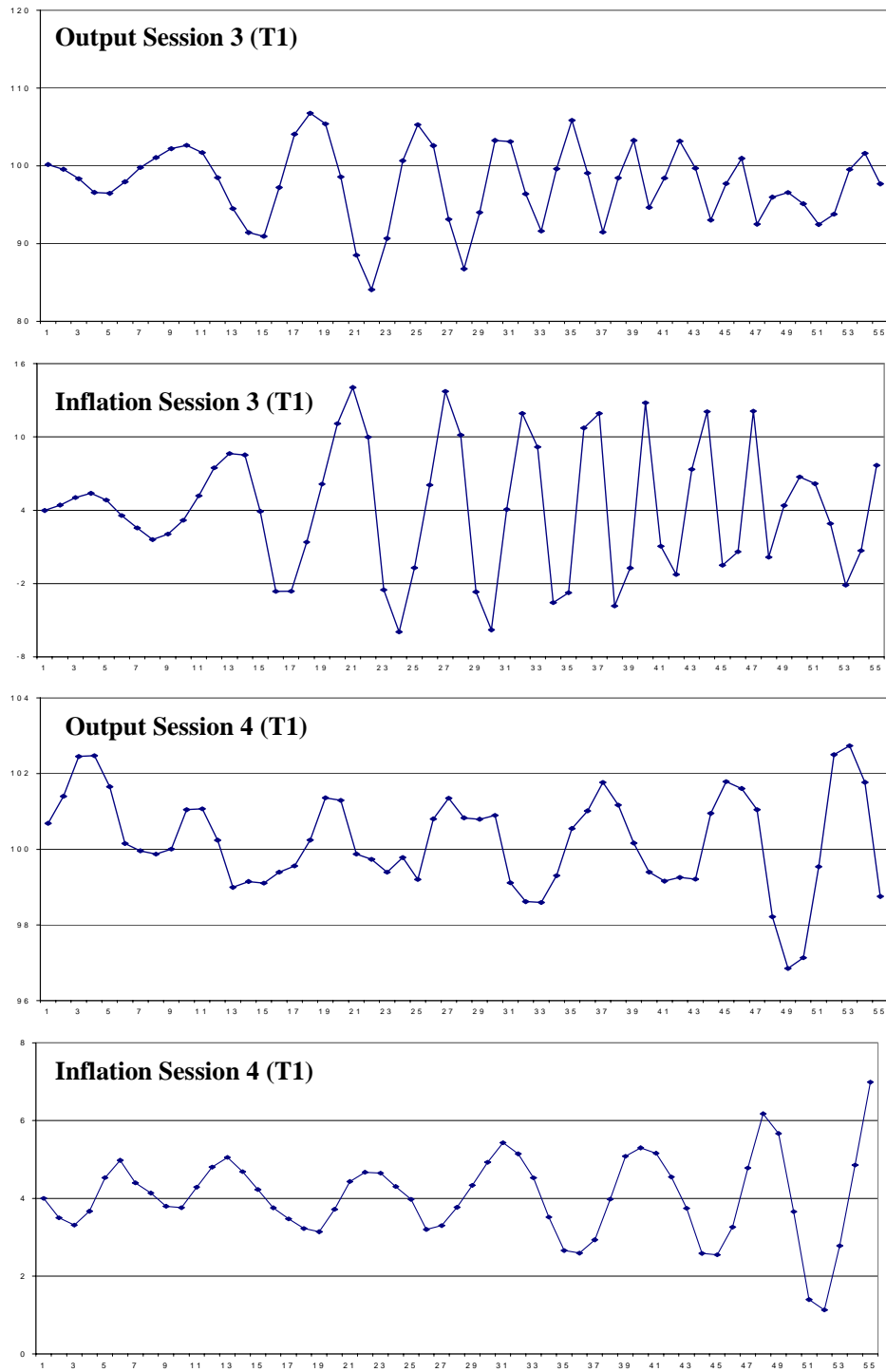


Figure 3: Output and Inflation in Selected Baseline Sessions



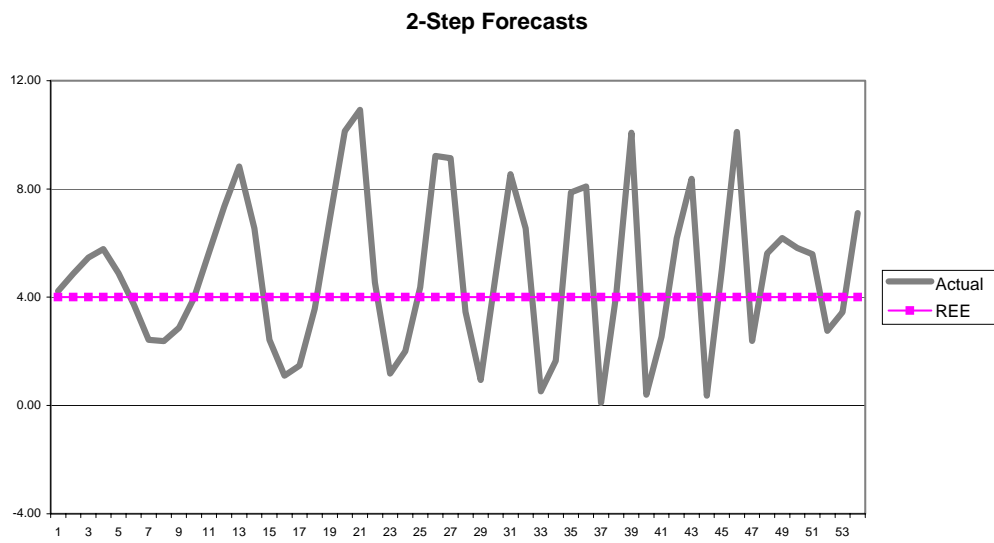
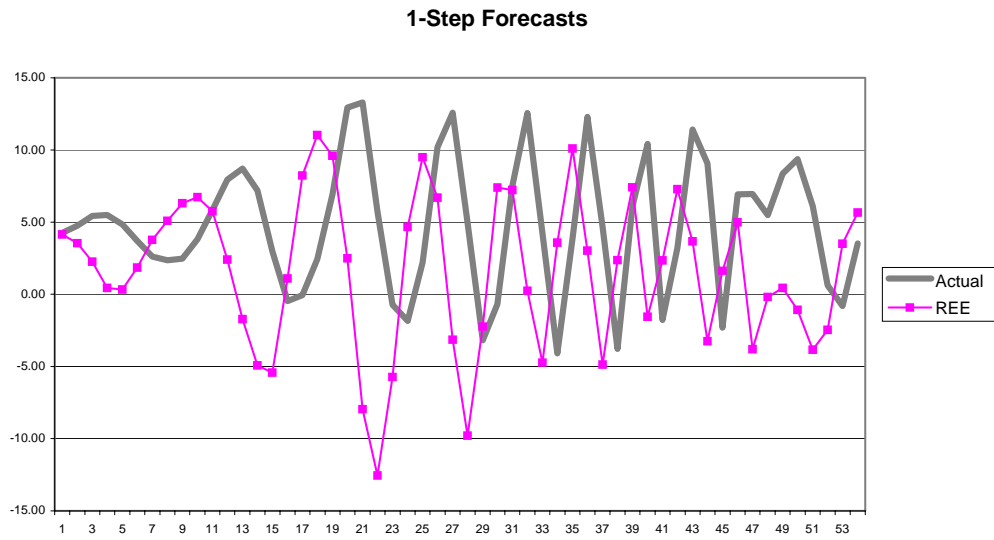


Figure 4: Actual Forecasts and RE Forecasts, Session 3 (T1)

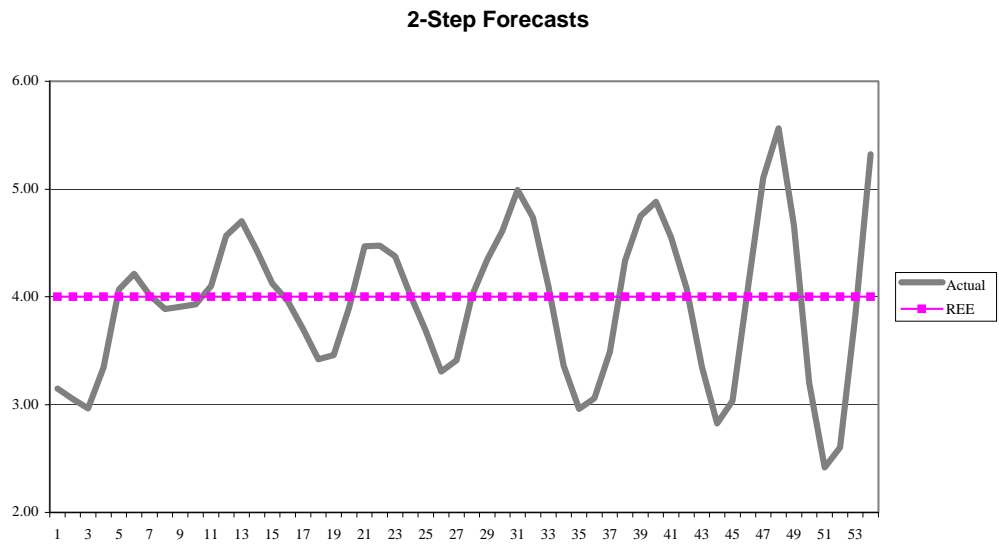
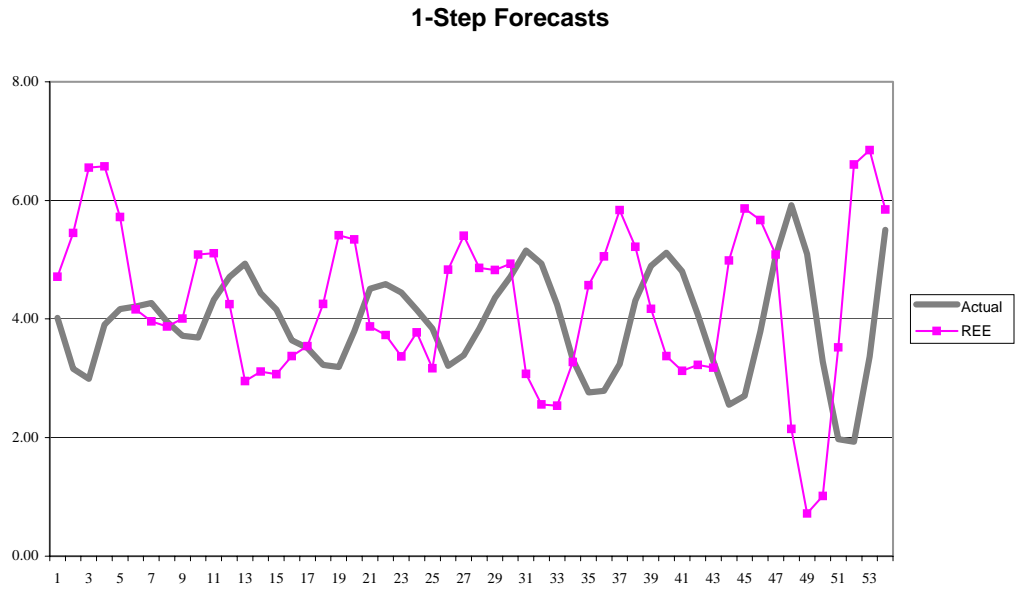


Figure 5: Actual Forecasts and RE Forecasts, Session 4 (T1)

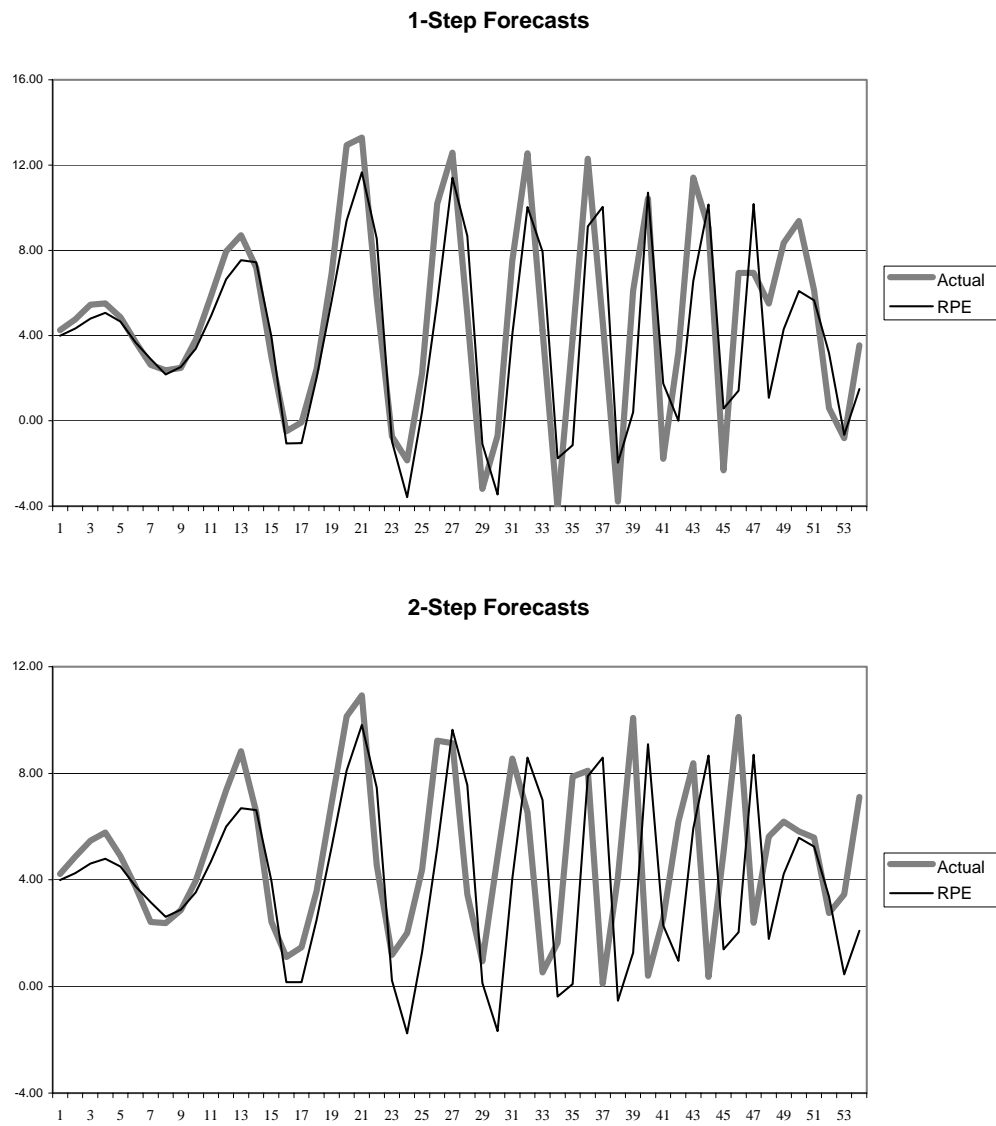


Figure 6: Actual Forecasts and Restricted Perceptions, Session 3 (T1)

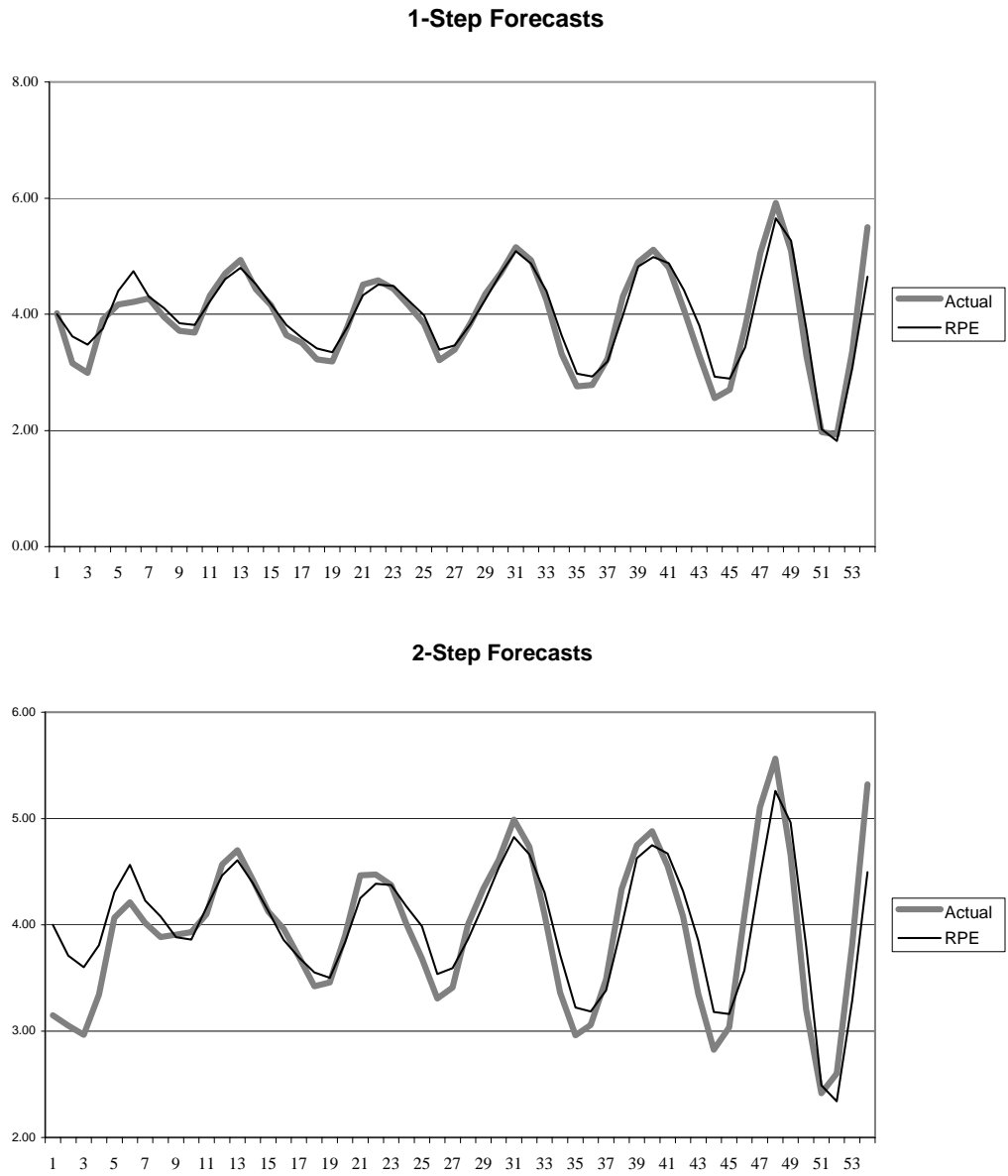
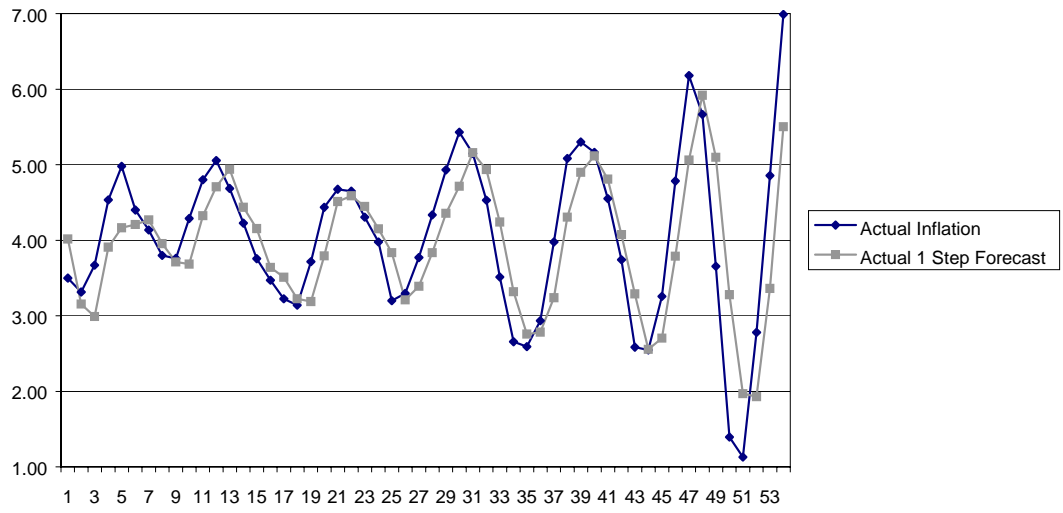


Figure 7: Actual Forecasts and Restricted Perceptions, Session 4 (T1)

### 1-Step Forecast



### 2-Step Forecast

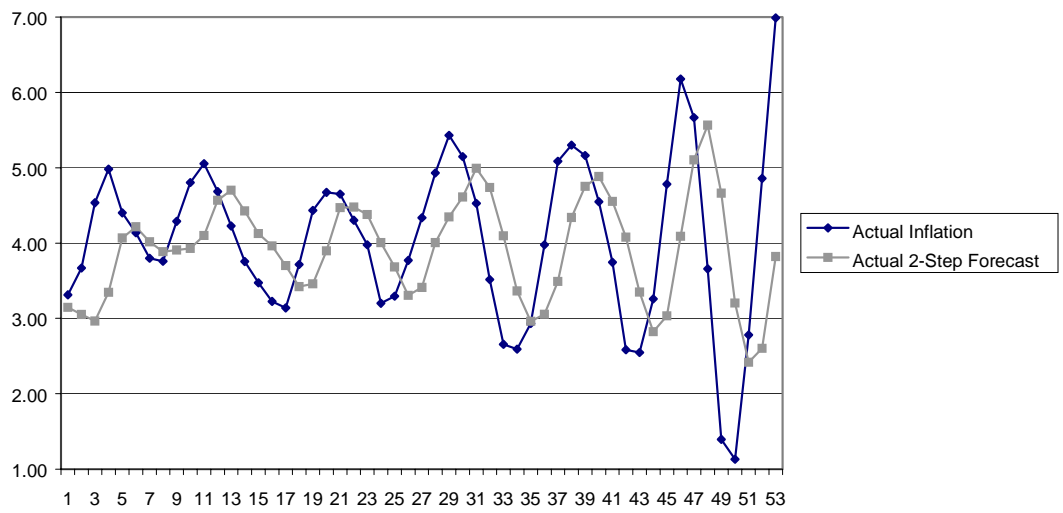
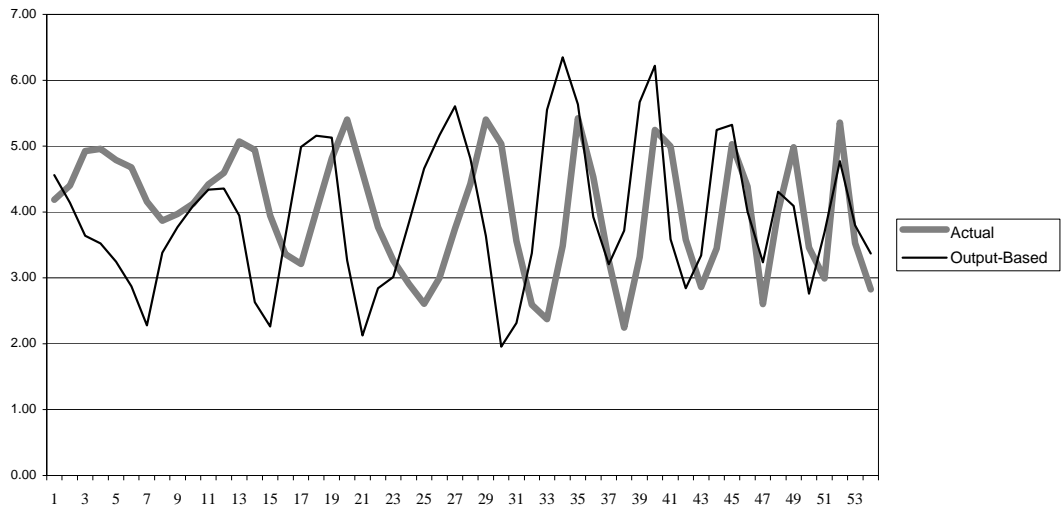


Figure 8: Actual and Predicted Inflation, Session 4 (T1)

**2-Step Forecasts, Session 1 (T1)**



**2-Step Forecasts, Session 3 (T1)**

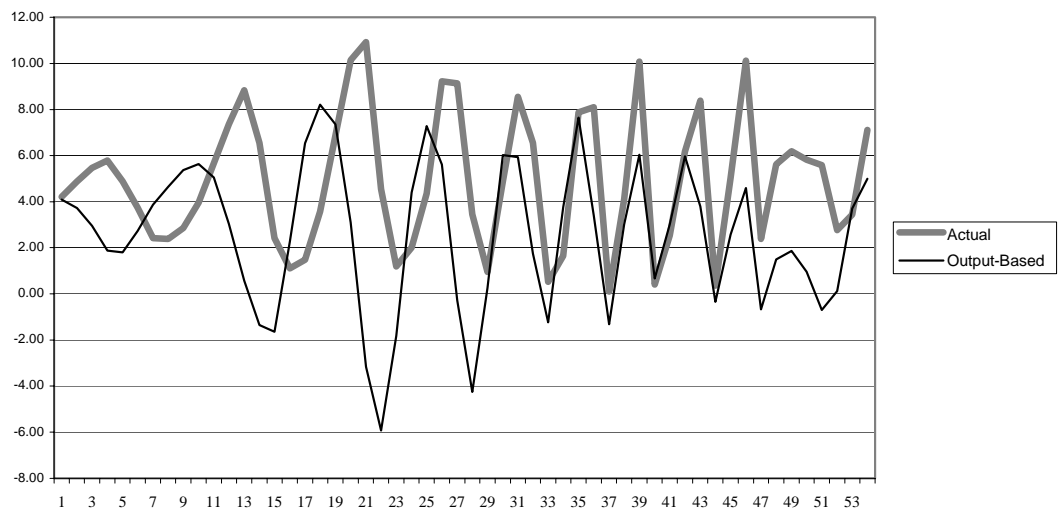
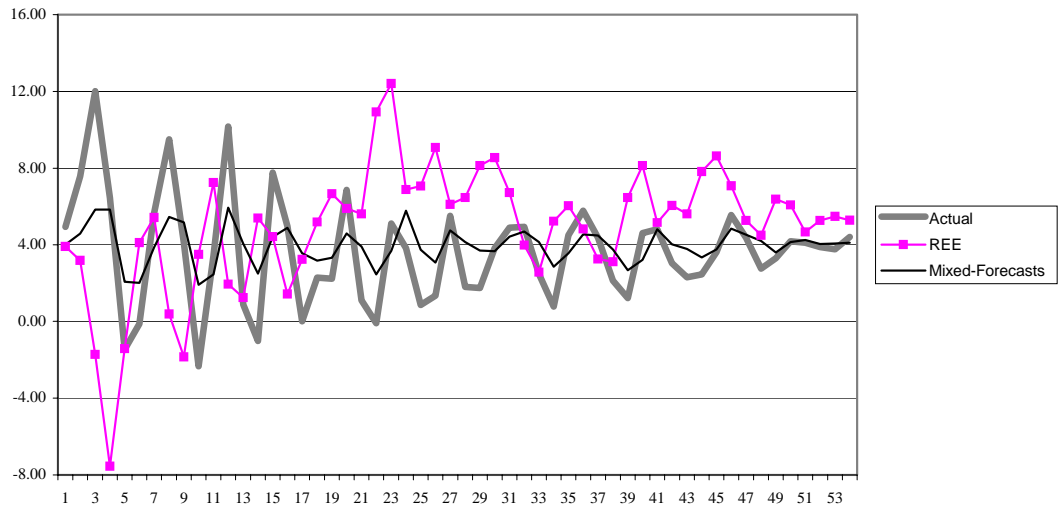


Figure 9: Output-Based 2-Step Forecasts

### 1-Step Forecasts



### 2-Step Forecasts

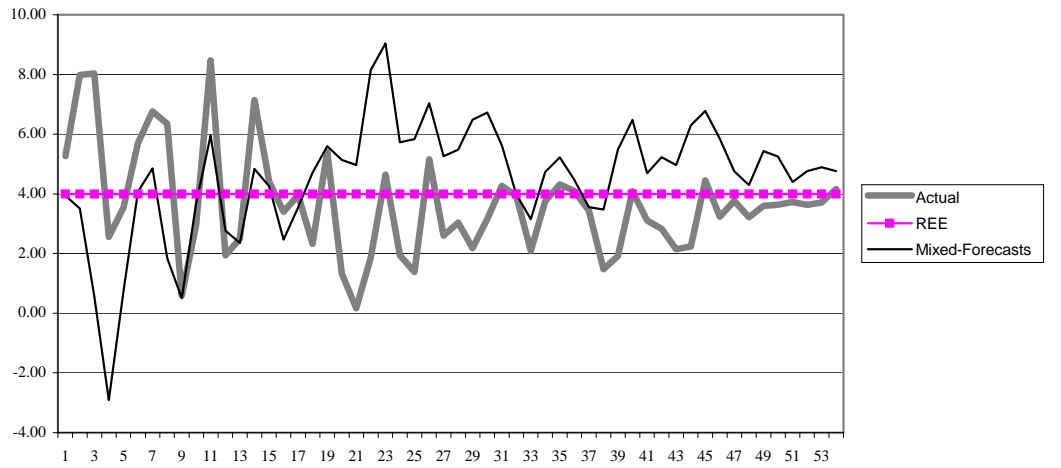


Figure 10: Sessions 3 (T2)

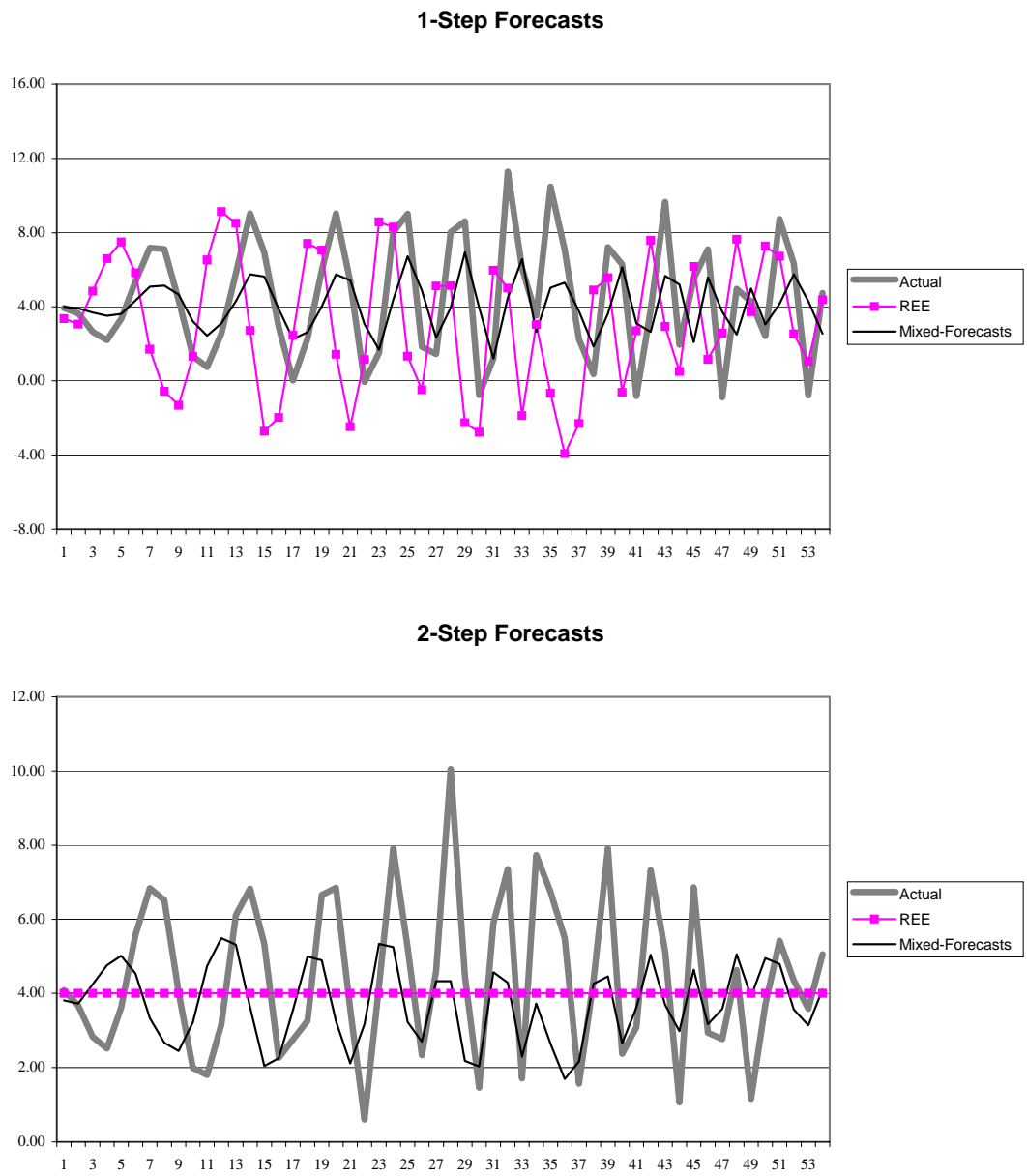


Figure 11: Session 4 (T2)



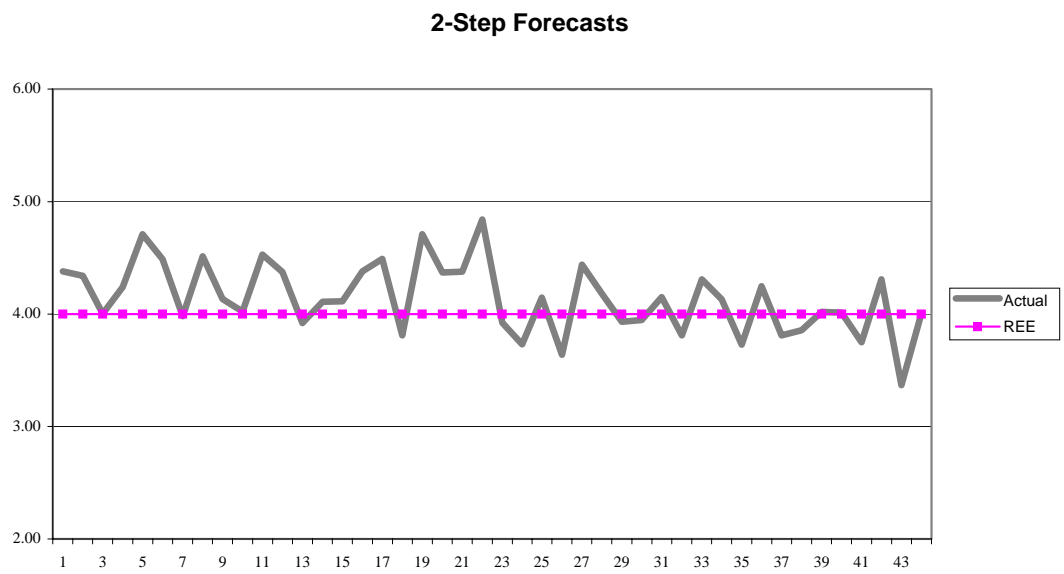
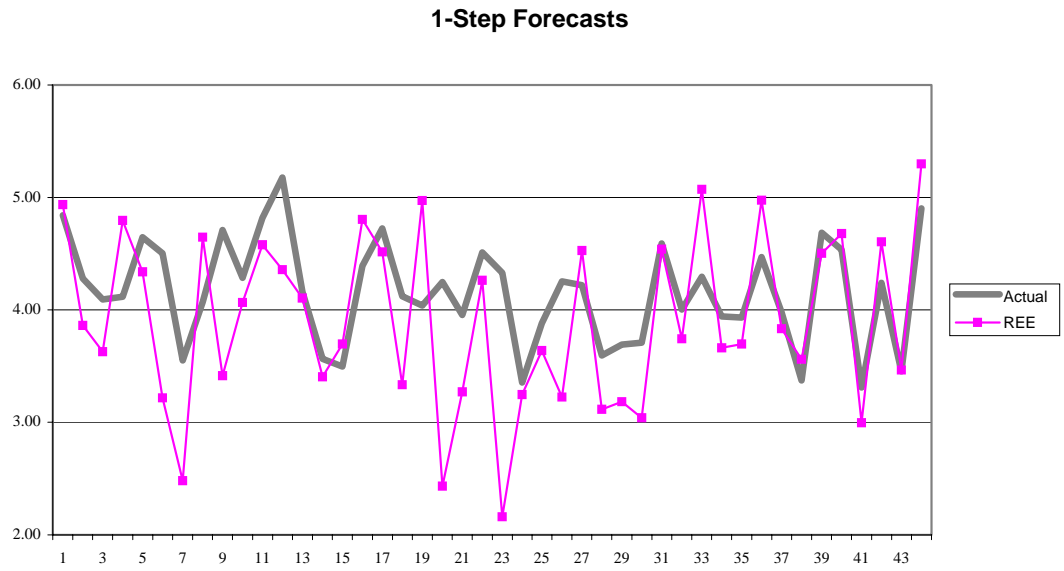


Figure 12: Session 5 (T1)

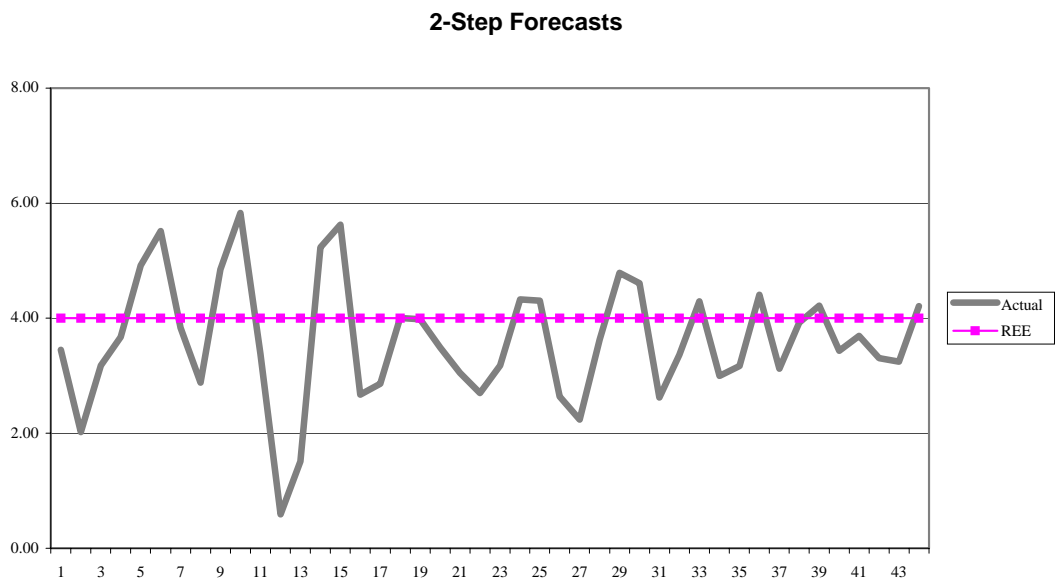
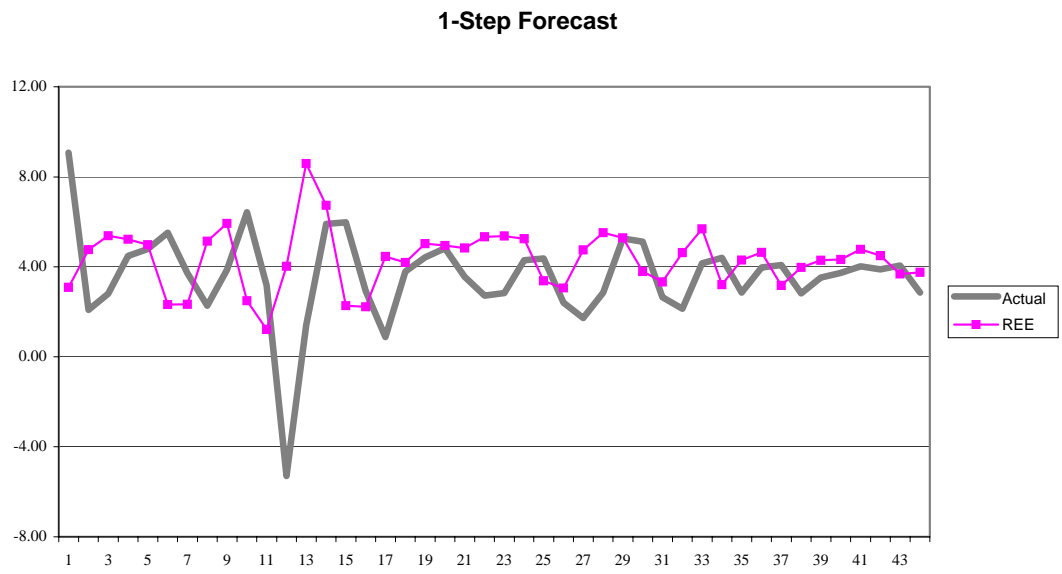


Figure 13: Session 6 (T1)

Table 1: Experimental Sessions and Treatments

	Treatment 1 (T1)	Treatment 2 (T2)	Location	Date
Session 1	high	-	Salerno, Italy	July 16, 2001
Session 2	high	-	Salerno, Italy	July 23, 2001
Session 3	high	high	Frankfurt, Germany	June 3, 2002
Session 4	high	high	Frankfurt, Germany	June 7, 2002
Session 5	low	-	Salerno, Italy	July 2, 2001
Session 6	low	-	Salerno, Italy	July 24, 2001

The highlighted treatments and sessions constitute the baseline experiments. 'low' indicates treatments where the elasticity of labor supply is set to  $\varepsilon = 1.0$ ; 'high' indicates treatments where the elasticity is set to  $\varepsilon = 2.0$ .

Table 2: Predicted and Actual Autocorrelations

	Autocorrelation	Output	Inflation
REE		0	0
RPE		0.70	0.76
Session 1 (T1)	0.53 (0.00)	0.38 (0.00)	
Session 2 (T1)	0.47 (0.00)	0.57 (0.00)	
Session 3 (T1)	0.46 (0.00)	0.23 (0.08)	
Session 4 (T1)	0.64 (0.00)	0.62 (0.00)	

Numbers in brackets are p-values for the Box-Ljung Q-statistic testing the null of no autocorrelation.

Table 3: Average Inflation Forecasts (Across Agents and Periods)

	Average 1-Step Forecast	Average 2-Step Forecast
Session 1 (T1)	4.03 (0.1584)	4.01 (0.1316)
Session 2 (T1)	3.89 (0.0972)	3.90 (0.0771)
Session 3 (T1)	4.75 (0.5138)	4.90 (0.3302)
Session 3 (T2)	3.64 (0.2801)	3.65 (0.2642)
Session 4 (T1)	3.95 (0.1481)	3.94 (0.1257)
Session 4 (T2)	4.48 (0.3104)	4.43 (0.2202)
Session 5 (T1)	4.16 (0.0605)	4.14 (0.0535)
Session 6 (T1)	3.58 (0.2284)	3.61 (0.1232)

Newey-West standard errors (3 lags) in parentheses.

Table 4: Baseline Treatments

1-Step Forecast	Share of RPE-Forecasters (vs. REE)	
	whole sample	last 20 periods
Session 1 (T1)	0.887 (0.0324)	0.747 (0.0427)
Session 2 (T1)	0.881 (0.0148)	0.855 (0.0265)
Session 3 (T1)	0.873 (0.0319)	0.778 (0.0530)
Session 4 (T1)	0.989 (0.0180)	0.969 (0.0228)
2-Step Forecast	Share of RPE-Forecasters (vs. REE)	
	whole sample	last 20 periods
Session 1 (T1)	0.534 (0.1477)	0.168 (0.1552)
Session 2 (T1)	0.694 (0.0501)	0.687 (0.0584)
Session 3 (T1)	0.212 (0.1896)	-0.436 (0.1435)
Session 4 (T1)	1.087 (0.0704)	1.069 (0.0677)
2-Step Forecast	Share of RPE-Forecasters (vs. Output-Based)	
	whole sample	last 20 periods
Session 1 (T1)	0.713 (0.0892)	0.354 (0.1025)
Session 2 (T1)	0.792 (0.0286)	0.744 (0.0407)
Session 3 (T1)	0.508 (0.1197)	0.068 (0.0947)
Session 4 (T1)	0.988 (0.0474)	0.928 (0.0334)

Newey-West standard errors (3 lags) in parentheses.

Table 5: Additional High Elasticity Treatments

1-Step Forecast	Share of Mixed-Forecasters (vs. REE)	
	whole sample	last 20 periods
Session 3 (T2)	1.14 (0.0646)	0.933 (0.0912)
Session 4 (T2)	0.950 (0.0792)	0.890 (0.1663)
2-Step Forecasts	Share of Mixed-Forecasters (vs. REE)	
	whole sample	last 20 periods
Session 3 (T2)	0.344 (0.3561)	0.449 (0.3468)
Session 4 (T2)	0.609 (0.2715)	0.812 (0.5870)
2-Step Forecast	Share of Mixed-Forecasters (vs. RPE)	
	whole sample	last 20 periods
Session 3 (T2)	0.835 (0.1119)	0.711 (0.1437)
Session 4 (T2)	0.795 (0.0887)	1.034 (0.1493)

Newey-West standard errors (3 lags) in parentheses. Session 3 included a dummy variable for period 20 to 30.

Table 6: Low vs. High Elasticity Treatments

1-Step Forecast	Coefficient on REE Forecasts	
	whole sample	last 20 periods
Session 5 (T1)	0.333 (0.0766)	0.717 (0.0927)
Session 6 (T1)	0.226 (0.0633)	0.216 (0.1055)
Session 1 (T1)	-0.136 (0.0468)	0.029 (0.1147)
Session 2 (T1)	-0.025 (0.0450)	-0.059 (0.0923)
Session 3 (T1)	-0.168 (0.0455)	-0.182 (0.0993)
Session 4 (T1)	-0.253 (0.0517)	-0.249 (0.0993)
Session 3 (T2)	-0.220 (0.0776)	0.000 (0.1295)
Session 4 (T2)	-0.0776 (0.0733)	-0.002 (0.1765)

Newey-West standard errors (3 lags) in parentheses.

Table 7: Low vs. High Elasticity Treatments

2-Step Forecast	(Actual - REE Forecast) <sup>2</sup>	
	whole sample	last 20 periods
Session 5 (T1)	0.116 (0.0244)	0.007 (0.0369)
Session 6 (T1)	1.260 (0.3941)	0.321 (0.0720)
Session 1 (T1)	0.787 (0.1044)	0.9839 (0.1207)
Session 2 (T1)	0.1945 (0.0332)	0.298 (0.0554)
Session 3 (T1)	9.123 (1.6336)	10.119 (2.2004)
Session 4 (T1)	0.493 (0.1106)	0.886 (0.1642)
Session 3 (T2)	3.307 (0.7934)	1.102 (0.4665)
Session 4 (T2)	4.672 (0.7331)	3.922 (0.8545)

Newey-West standard errors (3 lags) in parentheses.