Monetary Policy and Aggregate Volatility

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Abstract

Discretionary conduct of monetary stabilization policy can increase real and nominal aggregate volatility by arbitrary amounts when firms pay limited attention to aggregate shocks. A conservative central banker with stronger preference for price stability eliminates the commitment problem, thereby reduces output and price volatility and gives rise to a policy-induced 'Great Moderation'. Increased focus on price stability facilitates firms' information processing and aligns their expectations better with policy decisions. This 'coordination effect' reduces aggregate real and nominal volatility. Consistent with empirical evidence, the moderation manifests itself through reduced residual variance in vector autor regressions (VARs) involving macroeconomic variables.

Keywords: great moderation, optimal monetary policy, information processing frictions, output and price volatility.

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1 Introduction

The aim of this paper is to present a simple monetary policy model in which increased emphasis on price stability by a discretionary central bank can give rise to arbitrarily large reductions in the variance of aggregate output and inflation. Conversely, the model suggests that overly ambitious attempts to stabilize the real economy via discretionary monetary policy actions can result in very large increases in the volatility of real and nominal variables.

With regard to the first point, the model thus suggests the existence of a causal link between two major macroeconomic events that are widely believed to have taken place around 1980 in a number of developed economies: (1) following the inflation experience of the 1970's many central banks seem to have increasingly focused on insuring price stability.[1] In the U.S. this policy shift is typically associated with the appointment of Paul Volcker as chairman of the Federal Reserve and the subsequently implemented disinflation program; (2) the volatility of aggregate output and inflation has fallen significantly around the beginning of the 1980's, a fact generally referred to as the 'Great Moderation' and first documented in McConnell and Quiros (2000) and Blanchard and Simon (2001).

Conversely and with regard to the second point, the model suggests that the recently observed increase in aggregate real and nominal volatility may possibly be attributed to a renewed shift in the objectives of monetary policy towards greater stabilization of the real economy. Clearly, it is still too early to say whether such a shift has actually occurred and whether the recent volatility increase is more than a temporary phenomenon. The paper, therefore, focuses on the aggregate attenuation resulting from greater emphasis on price stability, although the proposed mechanism works also in the reverse direction.

The monetary model presented is a standard rational expectations model with maximizing firms and consumers. The key new feature of the model is that firms are assumed to face constraints on the amount of information they can process about aggregate shocks and about policy decisions. This follows recent work by Sims (2003) which stresses the scarcity of information in decision making, based on the observation that processing and incorporating information into decisions is not a costless process.[2] This paper emphasizes the information processing problems of price setting firms, as these appear of particular relevance for questions related to the conduct of monetary policy.[3]

The presence of information processing frictions implies that the quality of firms' information about their profit maximizing price is endogenous and depends, amongst other things, on the conduct of monetary policy. Specifically, monetary policies that give rise to large volatility of firms' profit maximizing price also make it harder for firms to track the precise value of their truly optimal price and thereby give rise to larger information processing errors. These

[1] Prominent proponents of this view include Clarida, Galì, and Gertler (2000), Cogley and Sargent (2001), and Orphanides and Williams (2005).

[2] See Moscarini (2004), Reis (2006*a*, 2006*b*), or Adam (2007) for applications.

[3] Mackowiak and Wiederholt (2008) have shown such frictions to be important for explaining observed pricing behavior by firms.

processing errors increase the variability of firms' information sets and lead to a misalignment between the private sector decisions and the actual policy stance. Since these misalignments are unpredictable for policymakers, they end up amplifying the nominal and real volatility in the economy.

In the present setting discretionary maximization of social welfare by the monetary authority is shown to generate excessively volatile monetary policy decisions compared to the fully optimal policy with commitment. Volatility of monetary policy induces volatility of profit maximizing prices and this leads - via the channels just described - to excessive real and nominal volatility in the aggregate economy. The commitment problem emerges because discretionary policy fails to incorporate the amount of information noise it generates: the variance of the information processing noise is a function of the *average* volatility of policy decisions in response to shocks, while the discretionary policy problem consists of determining the strength of the policy reaction to a *specific* shock realization; since the latter contributes little (nothing with continuous shock distributions) to the overall variance of policy, it is rational to ignore it under discretionary maximization.

The commitment problem is thereby more pronounced in economies in which firms can process information rather well because achieving any desired real effect then requires a larger amount of variation in the policy instrument.[4] Therefore, if over time firms become better at processing information, say due to technological progress in information and communications technologies, the response of discretionary monetary policy is to increase the variability of the policy instrument. This gives rise to increased aggregate variability and larger processing errors, which more than compensates the reduction in processing errors resulting from increased capacity to process information. This in turn suggests that the increased volatility of the 1970's in the U.S. when compared to the 1960's may be partly the result of an increasingly severe monetary commitment problem.

The paper shows that appointing a 'conservative central banker' à la Rogoff (1985), who place greater emphasis on price stability, reduces the volatility of policy decisions and allows - for an appropriate weight on price stability - to replicate optimal commitment policy via discretionary maximization.[5] With a conservative central banker the monetary policy response to shocks is less activist, which reduces the variance of firms' optimal price and thereby their processing errors. The resulting increased *coordination* between policy decisions and firms' pricing decisions is shown to unambiguously lower aggregate price and output volatility, independently of the model parameterization. This mechanism will be referred to as the 'coordination effect'.

The paper also analyzes the model-implied vector autoregressive (VAR) dynamics for output, prices and the monetary policy instrument. Interestingly, a marginal improvement in monetary policy (less activist policy) can result in no change of the auto-regressive coefficients - including those coefficients describing the VAR's 'policy equation' - but manifest itself via reduced variance of the

[4] In this paper the real effects of monetary policy are exclusively due to information processing constraints (information asymmetries).

[5] Vestin (2006) derives similary results within a sticky price economy.

VAR residuals. This suggests that it is well possible that some of the findings of the empirical VAR literature, e.g., Canova and Gambetti (2008), Primiceri (2005), or Sims and Zha (2006), are consistent with the notion that the Great Moderation is partly the result of improvements in monetary policy.

Even more strikingly, estimating VARs on model generated data from a monetary regime with and a monetary regime without a conservative central bank, and exchanging the (correctly identified) monetary reaction functions across the estimated VARs, results in unchanged behavior for the then resulting output and price process. This is the case, although in the model the volatility differences are exclusively due to a change in the conduct of monetary policy. This finding is of interest because empirical findings of this kind have occasionally been interpreted as suggesting that monetary policy is an unlikely explanation for the observed 'Great Moderation'.

More generally, the model suggests that the variability of private sector information sets (information processing errors) can be an important source of 'fundamental' shocks entering the residuals of empirical VARs and that the volatility of these information sets may be crucially influenced by the conduct of policy. This is consistent with the empirical findings in Galí and Gambetti (2008) who report that in the U.S. the observed volatility reduction is largely due to a substantial fall in the volatility of non-technology shocks.

Obviously, the model (trivially) predicts that lower price and output volatility can occur without any changes in monetary policy, e.g., following a reduction in the variance of standard shocks (shocks that are not processing errors). Therefore, the model is equally consistent with the notion that the findings of the empirical VAR literature are simply the result of reduced shock variance.

The paper is structured as follows. It starts in section 2 with a brief overview over the related literature. Section 3 then presents a simple static version of the model with imperfectly informed firms and derives a linear-quadratic approximation to the monetary policy problem. After introducing firms' information processing constraints in section 4, section 5 derives the monetary policy implications. In particular, it is shown how the presence of information processing constraints causes discretionary monetary policy to generate excessive aggregate volatility and how increased focus on price stability reduces volatility. Section 7 extends the static model to an infinite horizon economy. It derives and discusses the model-implied VAR dynamics for output, prices, and the monetary policy instrument and compares it to stylized facts documented in the empirical literature. A conclusion briefly summarizes. All proofs are contained in the web appendix to this article.

2 Related Literature

A number of mechanisms have been suggested in the literature through which improvements in the conduct of monetary policy may cause reduced output and price volatility, see Stock and Watson (2003) for an excellent survey.

Orphanides and Williams (2006, 2005) show that the usual trade-off between output and price volatility can disappear in a setting in which the private sector is perpetually learning about the dynamics of the economy by extrapolating from past economic behavior. More stable prices then reduce the volatility of the private sector's price expectations and thereby the overall volatility in the economy. Along a similar line, Branch et al. (2007) provide an explanation of the Great Moderation using a model in which the private sector can choose the level of attention and show how the model can give rise to high and low attention equilibria with high and low aggregate volatility, respectively.

Within the realm of rational expectations models, Clarida, Galí, and Gertler (2000) interpreted the 1970's as a period in which the monetary policy rule may have allowed for sunspot fluctuations, while from the start of the 1980's policy behavior created a determinate equilibrium. Surico and Benati (2007) explore the implications of this idea for the Great Moderation. Even without sunspot multiplicities, standard rational expectations models suggest that improvements in the conduct of monetary policy can give rise to reduced real and nominal variance. Clarida, Galí, and Gertler (2000), for instance, argue that a stronger monetary policy response to inflation fluctuations can reduce volatility. Boivin and Giannoni (2006) present an extended New Keynesian model in which a stronger monetary policy response to inflation reduces the variability of inflation and output following demand shocks. They conclude that a change in systematic policy together with a change in the mix of shocks may explain the observed great moderation in the US. The analysis in Stock and Watson (2003) suggests, however, that when using more elaborate sticky price models such as the Smets-Wouters (2007) model, New Keynesian models may have difficulties in attributing the reduced volatility of real output in the U.S. to the observable changes in the conduct of monetary policy.

The present model differs from the existing literature by assuming fully rational agents, by giving rise to a (locally) unique equilibrium prediction for all policy parameterizations, and by being able to generate arbitrarily large real and nominal volatility reductions following increased emphasis on price stability by discretionary monetary policy. It is worth emphasizing that the ability of the present model to generate the historically observed reduction of aggregate real and nominal volatility depends critically on the value one assigns to firms' processing capacity. Specifically, arbitrarily large volatility effects can be generated by assuming firms to be sufficiently efficient in processing information. At present, however, little can be said about what constitute reasonable calibrations for firms' processing capabilities. The quantification of the policy effects is therefore a subject that is left for future research.

3 The Basic Model

This section introduces a stylized monetary policy model and derives a linear-quadratic approximation to the optimal monetary policy problem. To simplify the exposition, it considers a static model and assume firms' information to be exogenous. Endogenous information sets will be introduced in section 4 when studying firms that optimally process information subject to processing constraints. Section 7 extends the setup to a fully dynamic model.

Importantly, the model below features two kinds of economic shocks, namely a shock that leads to socially desirable variation in private sector behavior, as well as a shock that induces the private sector to react in a socially undesirable way. This distinction will be important because the second kind of shock gives rise to a conflict between the private sector and the policymaker, while the first shock does not. Arguably, the precise source for the two kinds of shocks is not relevant for the results that follow.

Households The household sector is described by a representative consumer choosing aggregate consumption Y and labor supply L to maximize

$$\begin{aligned} & U(Y) - \nu V(L) \\ & s.t. \\ 0 = & WL + \Pi - T - PY \end{aligned} \tag{1}$$

where W denotes a competitive wage rate, Π monopoly profits from firms, T lump sum taxes, and P the price index of the aggregate consumption good. The parameter $\nu > 0$ is a stochastic labor supply shifter with $E[v] = 1$ and induces variations in the efficient labor supply and the level of output. Households are fully informed about all relevant aspects in the economy.[6] Furthermore, $U' > 0$, $U'' < 0$, $\lim_{Y \to \infty} U'(Y) = 0$, $V' > 0$, $V'' > 0$ and $V'(0) < U'(0)$.

Firms The supply side of the economy is characterized by a continuum of monopolistically competitive firms $i \in [0, 1]$ that can freely adjust prices but possess imperfect information about the aggregate shocks hitting the economy. Firm i produces an intermediate good Y^i with labor input L^i according to a linear production function of the form

$$Y^i = L^i$$

Intermediate goods enter into aggregate output Y according to a Dixit-Stiglitz aggregator

$$Y = \left(\int_{[0,1]} (Y^i)^{\frac{\theta-1}{\theta}} \mathrm{di} \right)^{\frac{\theta}{\theta-1}} \tag{2}$$

where the demand elasticity $\theta > 1$ is stochastic with mean $E[\theta] = \overline{\theta}$. Variations in θ will induce socially undesirable variations in the price mark-up charged by the monopolistically competitive firms,[7] and will give rise to a conflict between firms and monetary policy.

Let $Y^i(P^i/P)$ denote household's utility-maximizing demand for product Y^i induced by (2) when firm i charges price P^i and the price for the aggregate good is P. The profit maximization problem of firm i is then given by

$$\max_{P^i} E\left[U'(Y)\left((1+\tau)P^i Y^i(P^i/P) - WY^i(P^i/P)\right)|I\right] \tag{3}$$

[6] Households, however, would only need to know the wage rate, the prices charged by firms, and their income.

[7] Mark-up variations are inefficient because an output subsidy for firms is assumed to eliminate the average mark-up charged by these firms, see below.

where τ denotes an output subsidy that eliminates the average mark-up charged by monopolistically competitive firms and I the firm's information set, which contains information about the labor supply shock ν, the demand shock θ, and monetary policy decisions. For simplicity it is assumed in equation (3) that all firms possess the same information set I. It is shown in section 6.3 that this assumption is not essential for the results that follow. All that is required is that firms share *some* common (noisy) piece of information about fundamentals.

Monetary Policy The monetary policymaker is supposed to choose nominal demand and maximize the utility of the representative agent (1). To simplify the analysis, a linear-quadratic approximation to the optimal policy problem is studied, i.e., a quadratic approximation to social welfare and a linear approximation to the implementability conditions characterizing optimal private sector behavior.

Appendix A.1 shows that when the output subsidy τ eliminates the average price mark-up charged by firms a quadratic approximation of the representative agent's utility function (1) is given by

$$-(y - y_n)^2 \tag{4}$$

where $(y - y_n)$ denotes the output gap. In particular, y is the average output across firms, i.e., $y = \int y(j)\mathrm{d}j$, and y_n the efficient output level.[8] The latter depends on the labor supply shock ν only and is assumed normal, i.e., $y_n \sim N(0, \sigma_y^2)$. The remaining part of the paper refers to y_n as a *natural rate shock*.

Appendix A.1 also derives the linear approximation to firms' optimal price setting behavior implied by problem (3):

$$p(i) = E[w + \varepsilon | I] \tag{5}$$

The previous equation describes the profit maximizing behavior of firm i conditional on available information I.[9] The profit maximizing price $p(i)$ can be expressed as a mark-up ε over the nominal wage w. The optimal mark-up is given by

$$\varepsilon \sim -(\theta - \overline{\theta}).$$

Firms wish to charge a higher mark-up ($\varepsilon > 0$) whenever the price elasticity of demand θ falls below its mean $\overline{\theta}$. A positive value of ε thus reflects the fact that product demand has become less price-sensitive. We refer to ε as a *mark-up shock*.[10] It is assumed independent of the natural rate shock y_n and $\varepsilon \sim N(0, \sigma_\varepsilon^2)$. The nominal wage w can be expressed as

$$w = p + \xi(y - y_n)$$

It depends on output y, which captures total labor demand, on the natural rate shock y_n, capturing shifts in labor supply, and on the price level p, which is

[8] Lower case letters indicate that variables are expressed as percentage deviations from steady state values.

[9] It also incorporates the implementability conditions implied by the optimal labor-leisure choice of households, see appendix A.1 for details.

[10] This follows Woodford (2003). Galí et al. (1999) refer to it as a 'cost-push shock'.

influenced by monetary policy decisions. The parameter $\xi > 0$ thereby indicates the sensitivity of firms' prices to the output gap and is given by

$$\xi = -\frac{U''(\overline{Y})\overline{Y}}{U'(\overline{Y})} + \frac{V''(\overline{Y})\overline{Y}}{V'(\overline{Y})}$$

where $\overline{Y}$ denotes the steady state output level. Summarizing the previous results we have

$$p(i) = E\left[p + \xi(y - y_n) + \varepsilon | I\right] \tag{6}$$

Equation (6) is essentially a Lucas supply curve with the added feature that the information set I will be endogenous to the model.

The central bank controls nominal spending q, which is defined as

$$q = y + p. \tag{7}$$

Combining this with equation (6), one obtains that in a symmetric price setting equilibrium

$$p = E\left[p^* | I\right] \tag{8}$$

where p^* denotes the optimal price under perfect information

$$p^* = q - y_n + \frac{1}{\xi}\varepsilon \tag{9}$$

The price p^* depends on the fundamental shocks and on monetary policy. As one would expect the optimal price increases (decreases) one-for-one with nominal demand (the efficient output level) and increases in response to mark-up shocks. The response to mark-up shocks is more pronounced the smaller is ξ. Low values of ξ indicate that marginal costs react only little to output. Therefore, to obtain the desired increase in mark-ups following a positive shock to ε, a larger real contraction is required, i.e., a larger increase in nominal price ceteris paribus.

Timing of Events and Information Sets Figure 1 illustrates the sequence of events taking place in the economy. First, the stochastic disturbances (y_n, ε) realize. In the baseline case the central bank can observe these disturbances perfectly (although this assumption is relaxed later on) and then determines the desired level of nominal demand. Thereafter, firms process information about the shock realizations and the central bank's policy choice, as is explained in the next section, and simultaneously determine product prices. Then, consumers observe the prices posted by firms and demand products while firms simultaneously hire the labor necessary to satisfy product demand at the posted price. Finally, production and consumption take place.

As should be clear from figure 1, firms do not perfectly observe their nominal marginal costs at the time they set their prices. This is so because the labor market clears only after product prices have been determined. Since nominal wages and the optimal mark-up over nominal wages depend on the exogenous disturbances and on the monetary policy decisions, firms have an incentive to process information about these variables before determining their prices.

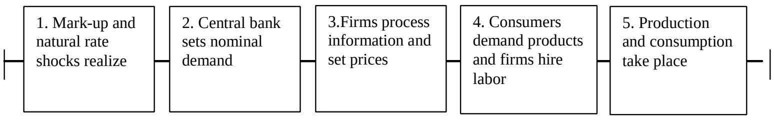

Figure 1: Sequence of events

Monetary Policy: Discretion versus Commitment Summarizing the results derived above, the monetary policy problem consists of choosing q so as to maximize (4) subject to (7)-(9). A discretionary policymaker thereby determines policy only after the mark-up and natural rate shocks have realized, i.e., at the time of implementation (stage two in figure 1). A policymaker who could commit would determine a fully state contingent policy $q(\varepsilon, y_n)$ before shocks realize and simply implement the predetermined policy choices once economic disturbances have materialized. As will be shown, this makes a crucial difference for the policy outcome.

4 Optimal Information Processing

This section consider firms that in addition to choosing prices also determine what information to process. Firms choose prices and information structures to maximize their profits but face a constraint on the total amount of information that can be processed each period, as in Sims (2003). The section first derives results formally and then offers a more intuitive interpretation of the findings. The aim of this section is to show how - in a setting with information processing constraints - monetary policy gives rise to a ‘coordination effect’.

A quadratic approximation of the firm’s profit is given by

$$-E\left[(p-p^*)^2 |I\right] \tag{10}$$

The firm chooses p and I so as to maximize (10) subject to an information processing constraint

$$H(p^*) - H(p^*|I) < K \tag{11}$$

where $H(p^*)$ denotes the entropy about p^* before processing information and $H(p^*|I)$ the entropy after information processing.[11] Intuitively, entropy is a measure of the uncertainty about a random variable. Stated in these terms, the processing constraint (11) provides a bound $K \in [0, \infty]$ on the maximum uncertainty reduction about p^* that can be achieved by processing information. The bound K is thereby measured in ‘bits’, i.e., number of zeros and ones, per unit of time. For $K = 0$ no uncertainty reduction is possible, i.e., firms cannot process information at all, while for $K \to \infty$ firms process information perfectly.

[11] The entropy $H(X)$ of a continous random variable X is defined as $H(X) = -\int {}_2\log(x)p(x)dx$ where $p(x)$ is the probability density function of X and where the convention is to take ${}_2\log(x)p(x) = 0$ when $p(x) = 0$.

For a given information structure I, the optimal price choice is $p = E[p^*|I]$ so that the expected loss associated with information structure I is equal to $Var(p^*|I)$. Choosing an optimal information structure thus amounts to minimizing $Var(p^*|I)$ subject to the constraint that the conditional entropy $H(p^*|I)$ cannot fall below the threshold defined by the processing constraint (11). Shannon (1948) shows that Gaussian variables minimize the conditional variance for a given entropy level.[12] It is therefore optimal to choose $p^*|I$ to be Gaussian so as to achieve an infimum value for $Var(p^*|I)$. Since p^* is Gaussian (due to the assumption of Gaussian fundamental shocks), the posterior $p^*|I$ is also Gaussian, provided the information I available to firms takes the form a signal s with representation

$$s = p^* + \eta, \tag{12}$$

where $\eta \sim N(0, \sigma_\eta^2)$ denotes the firm's information processing noise, which is independent of all other random variables in the model. The variance of the processing noise η is thereby given by[13]

$$\sigma_\eta^2 = \frac{1}{2^{2K} - 1} Var(p^*). \tag{13}$$

This is the infimum variance such that the information structure $I = \{s\}$ still satisfies the constraint (11). In particular, choosing a lower variance would imply that firms process more than K bits of information. As one would expect, the processing noise falls and the information structure becomes more informative, if firms' processing capacity K increases.

Given the optimal information structure, the firm's optimal price is then

$$\begin{aligned} p &= E[p^*|I] \\ &= E[p^*|s] \\ &= k \cdot s, \end{aligned} \tag{14}$$

where the Kalman gain $k \in [0, 1]$ is

$$k = \frac{Var(p^*)}{Var(p^*) + \sigma_\eta^2} = (1 - 2^{-2K}). \tag{15}$$

The Kalman gain k is a useful summary statistic indicating how well agents can process information about their environment. For $k = 0$ firms receive no information since $\sigma_\eta^2 = \infty$. Conversely, for $k = 1$ firms observe perfectly since $\sigma_\eta^2 = 0$. At intermediate values of k the variance of the observation noise η is positive and decreases with k.

Coordination Effect The expression for the variance of the processing noise in equation (13) shows that a more variable full information price p^* causes firms' to make larger processing errors. Intuitively, this occurs because

[12] Shannon solves the dual problem of maximizing entropy for a given variance.

[13] This follows from equation (11), the fact that the entropy of a Gaussian random variable is equal to one half its log variance plus a constant, and the updating formula for the variance of normal variables, i.e., $Var(p^*|s) = Var(p^*) - Var(p^*)^2/(Var(p^*) + \sigma_\eta^2)$.

information about a more variable environment is harder to track for any given capacity to process information. Since the noisy signal s enters with proportionality factor k into firms' prices, see equation (13), a more variable full information price causes firms' optimal price choice p to increasingly deviate from the full information price. This is formally summarized in the subsequent result:

Lemma 1 *A more variable optimal price p^* increases firms' pricing errors:*

$$Var(p - p^*) = (1 - k)\, Var(p^*)$$

Clearly, the variability of the full information price p^* depends on the conduct of monetary policy, see equation (9). Therefore, if monetary policy causes the full information price p^* to be very volatile, firms' will make larger processing errors, implying that firms' price choices will be less predictable for the monetary policymaker.[14] The endogeneity of the information structure thus suggests that monetary policy influences the economy along a new margin: by making p^* less variable, policy causes the private sectors' price choices to become more predictable, i.e., better coordinated around p^*. I call this attenuation the **'coordination effect'**.

Importantly, the coordination effect not only reduces unpredictable movements in prices, but also unpredictable movements in the output gap. From the definition $y = q - p$ and the results about firms' optimal price and information choice derived above one obtains

$$y - y_n = \left[(1 - k)\, q - (1 - k)y_n - \frac{k}{\xi}\varepsilon\right] - k\eta \tag{16}$$

Smaller information processing errors η thus also reduce the variance of unpredictable output gap movements. Whether this also implies a reduced overall volatility of the output gap depends on how the considered policy shift affects the volatility of the terms in the square bracket in equation (16). This issue is investigated in the next section.

5 Policy Implications of Processing Limitations

This section shows how the presence of the 'coordination effect' discussed in the previous section influences monetary policy decisions and economic outcomes.

Under discretionary policymaking it turns out optimal to simply ignore the coordination effect, implying that firms' profit-maximizing prices are rather volatile and their processing errors correspondingly large. This is shown to result in suboptimally high real and nominal volatility, when compared to the outcome under commitment. The volatility effects of discretionary policy are thereby particularly pronounced when firms a very efficient at processing information. It is then shown how the appointment of a 'conservative central banker', as in Rogoff (1985) can result in a large drop in real and nominal aggregate volatility.[15]

[14] The processing error η is unpredictable for policymakers as it realizes after monetary policy has been set, see section 3.

[15] A 'conservative central banker' is a policymaker who places more weight on achieving price stability than suggested by the social welfare criterion.

Throughout this section it is assumed that policymakers observe fundamental shocks without noise (have perfect capacity to process information) and that firms' processing capacity is exogenously given, i.e., independent of the way policy is conducted. Both assumptions will be relaxed in section 6 where it is shown that the main results carry over to these more general settings.

5.1 Discretionary Policy Activism

This section considers the volatility implications arising from discretionary policymaking. Summarizing results derived in sections 3 and 4, the discretionary policy problem can be expressed as

$$\max_{q} -E[(y-y_n)^2|\varepsilon, y_n] \tag{17}$$

$$s.t. :$$

$$y - y_n = \underbrace{(1-k)\,q}_{\text{standard effect}} - (1-k)y_n - \frac{k}{\xi}\varepsilon - \underbrace{k\eta}_{\text{coordination effect}} \tag{18}$$

with equation (18) describing the behavior of the output gap as a function of the policy choice, the fundamental shocks, and the realization of the processing error. Monetary policy affects the output gap in equation (18) trough 'standard effects' and via the 'coordination effect'.

The 'standard effect' of monetary policy is that arising in traditional imperfect information models, e.g., Lucas (1972, 1973). It predicts that an increase in nominal demand by one unit moves output (and the output gap) by $(1-k)$ units because the presence of processing constraints implies that firms do not fully perceive movements in nominal demand. These standard effects of policy can be used to amplify or dampen the effects of natural rate and mark-up shocks entering on the r.h.s. of equation (18). Note that they cannot be used to eliminate the effects of the processing error η, which only realizes after monetary policy has been determined.

Monetary policy also influences the behavior of the output gap through the 'coordination effect'. From the previous sections it follows that the variance of the coordination error is

$$\sigma_\eta^2 = \frac{1-k}{k} Var(p^*) \tag{19}$$

where the full information optimal price is given by

$$p^* = q - y_n + \frac{1}{\xi}\varepsilon \tag{20}$$

Importantly, the discretionary policy reaction to economic disturbances is determined at the time of implementation, i.e., only after the shocks (ε, y_n) have realized. Therefore, a discretionary policymaker can safely ignore the coordination effects of its current policy decision, i.e., can treat the variance of the processing error σ_η^2 in the policy problem (17) as given. This is rational because the policy reaction to a particular shock realization contributes little (nil with continuous shock distribution) to the ex-ante variability of the full information price p^* and thus to the coordination error.

The following proposition shows that ignoring the coordination effect can have stark implications:[16]

[16] This results follows directly from the first order conditions of problem (17).

Proposition 2 *In a rational expectations equilibrium with optimal information processing by firms, optimal discretionary policy is*

$$q = \frac{k}{(1-k)}\frac{1}{\xi}\varepsilon + y_n \tag{21}$$

The implied variance of the full information price is

$$p^* = \frac{1}{(1-k)}\frac{1}{\xi}\varepsilon \tag{22}$$

and the variances of the output gap and the price level are

$$\begin{aligned} E[(y-y_n)^2] &= \frac{k}{(1-k)\xi^2}\sigma_\varepsilon^2 \\ E[p^2] &= \frac{k}{(1-k)^2\,\xi^2}\sigma_\varepsilon^2 \end{aligned} \tag{23}$$

The proposition shows that aggregate nominal and real volatility increases without bound as firms become better and better at processing information, i.e., as k approaches 1. This occurs although improved information processing capabilities by firms reduces - ceteris paribus - their processing errors.

To understand this finding consider the case of a positive mark-up shock. This shock causes firms' desired mark-up to increase and results in a socially inefficient price increase. To avoid the corresponding drop in output, discretionary policy finds it optimal to nominally accommodate the shock. When firms are very efficient in processing information, the required amount of monetary accommodation has to be very large and this strongly increases the variability of firms' optimal price p^*: nominal accommodation generates an additional incentive for firms to increase their prices in response to positive mark-up shocks. More efficient information processing by firms therefore generates - through the induced monetary policy reactions - a very variable optimal price p^* and correspondingly large processing errors. These lead to higher nominal and real volatility which increases without bound as $k \to 1$.

Conversely, if firms are not very efficient in processing information (k close to zero) they fail to perceive the mark-up fluctuations, so that these shocks do not lead to socially inefficient price increases. There is then no need to nominally accommodate.

Finally, note that by nominally accommodating the natural rate shocks, monetary policy stabilizes at the same time the full information price p^*, see equation (20), and the output gap through the standard effects of monetary policy, see equation (18). Natural rate shocks move the efficient output level but do not give rise to a conflict between social and private incentives. Therefore, ignoring the coordination effect under discretionary policymaking entails no efficiency loss in the policy reaction to these shocks.

It is instructive to compare the discretionary policy outcome to fully optimal policy under commitment. With commitment the policymaker would determine *fully contingent policies before shocks realize* and take into account the 'coordination effect', i.e., the fact that the policy reaction to a specific shocks realization has implications for firms' uncertainty in response to all other conceivable shock realizations. The commitment problem is given by

$$\max_q -E[(y - y_n)^2]$$
$$s.t.: \text{ equations (18),(19) and (20)}$$

and the first order conditions of this problem imply the following result:

Proposition 3 *In a rational expectations equilibrium with optimal information processing by firms, optimal monetary policy under commitment is to set*

$$q = y_n \tag{24}$$

The implied variability of the output gap and prices are

$$E[(y - y_n)^2] = \frac{k}{\xi^2}\sigma_\varepsilon^2 \tag{25}$$
$$E[p^2] = \frac{k}{\xi^2}\sigma_\varepsilon^2$$

The previous proposition shows that with fully optimal policy, aggregate variability increases only linearly with the processing index k.[17] In particular, with commitment it turns out optimal *not to nominally accommodate mark-up shocks*. This is optimal so as to avoid the information processing errors induced by nominal accommodation.

From propositions 2 and 3 it is clear that discretionary policy approaches the commitment solution as $k \to 0$. For low levels of firms' processing capacity, lack of monetary commitment would thus not entail large utility costs due to increased aggregate volatility. However, if the economy is characterized by an increase in firms' processing capacity over time, a scenario that is not implausible given the vast advancements in information and communication technologies over time, a possible monetary commitment problem would become increasingly apparent. The increase in aggregate volatility in the 1970's experienced in a number of developed countries might thus partly be attributed to a monetary commitment problem which became increasingly apparent because firms became better and better at processing aggregate information. One possible interpretation of the monetary policy regime shift in the U.S. and a number of other countries in late 1970's and the beginning of the 1980's is thus that changes in the economic environment forced policymakers to actively look for a solution that would avoid the increasingly suboptimal economic outcomes induced by discretionary monetary policymaking. This issue is discussed in the next section.

[17] The slightly non-intuitive implication that output gap variability increases, i.e., welfare decreases, when firms can process information better (larger values of k) emerges because increased processing capacity has two opposing effects: on the one hand, it reduces the size of processing errors, which enhances welfare; on the other hand, it allows firms to observe aggregate mark-up shocks better, which decreases welfare. The latter effect dominates and causes output and price variability to linearly increase with firms' ability to process information (k).

5.2 Increased Focus on Price Stability

Following the experience of the 1970's, monetary policy in the U.S. and in a number of OECD countries started to refocus on the goal of achieving price stability. In the U.S. this policy shift is invariably associated with the appointment of Paul Volcker as chairman of the Federal Reserve Bank. The question asked in this section is whether in the proposed model such a shift in the monetary policy orientation may also give rise to a subsequent 'Great Moderation' in aggregate and nominal volatilities.

In the model, the actual price level p and the full information price p^* are linked via firms' information processing constraint. This indeed suggests that a discretionary policymaker which seeks to stabilize the price level will also implicitly stabilize p^*, thereby partly endogenize the 'coordination effect' of policy, and ultimately cause aggregate real and nominal volatility to fall.

To study this issue, consider a discretionary but 'conservative' central banker, as in Rogoff (1985), who is maximizing

$$\max -E\left[(y-y_n)^2 + \omega p^2 | \varepsilon, y_n\right] \tag{26}$$

$$s.t.$$

$$p = k\left(q - y_n + \frac{1}{\xi}\varepsilon + \eta\right) \tag{27}$$

$$y - y_n = (1-k)\,q - (1-k)y_n - \frac{k}{\xi}\varepsilon - k\eta$$

$$\sigma_\eta^2 \text{ given}$$

where $\omega \geq 0$ is the weight attached to price stability in the objective function, i.e., a parameter capturing the degree of 'monetary conservatism'. Since the policymaker continues to act under discretion, the variance of the processing error continues to be considered independent of policy decisions. The lemma below summarizes the main results.

Lemma 4 *Consider discretionary monetary policy with weight $\omega \geq 0$ on stabilizing prices. The variance of the output gap and prices is decreasing in ω for $0 \leq \omega < \frac{1-k}{k}$. For $\omega = \frac{1-k}{k}$ discretionary monetary policy is identical to the policy under commitment.*

Independently of the precise model parameterization, increased focus on price stability by a discretionary central bank reduces aggregate price and output gap volatility. Moreover, for a sufficiently high weight on the price stability goal, discretionary and conservative monetary policy replicates the optimal policy under commitment. The latter shows that the appointment of a conservative central banker can result in a large reduction of aggregate nominal and real volatility, i.e., give rise to a policy-induced 'Great Moderation' effect. It follows from propositions 2 and 3 that the volatility reduction associated with such a shift is particularly pronounced if firms can process information very well and becomes arbitrarily large as firms' processing capacity $k \to 1$. The present model is thus able to attribute arbitrary large changes in real and nominal volatility to changes in the conduct of monetary policy.

6 Robustness

This section shows that the previous findings are robust with respect to a number of important extensions. It relaxes the assumption that the central bank can process information about shocks perfectly, studies the effects of endogenizing firms' choice of processing capacity, and the effects of introducing idiosyncratic elements in firms' processing errors.

6.1 Central Bank Processing Limitations

This section considers a central bank facing information processing limitations of the same kind as previously introduced for firms. In particular, the central bank's choice for nominal demand q is now assumed to be subject to an information flow constraint

$$H(q^*) - H(q^*|q) < K^{CB}$$

where $K^{CB} \geq 0$ denotes the central bank's capacity to process information (expressed in bits per unit of time) and

$$q^* = a\varepsilon_t + by_{n,t} \tag{28}$$

the optimal decision under perfect central bank information. The coefficients a and b remain to be determined. As with firms, the central bank's optimal choice of information structure is a signal of the form

$$s^{CB} = q^* + \eta^{CB}$$

where $\eta^{CB} \sim N(0, \sigma^2_{\eta,CB})$ is a processing error limiting the information about q^* contained in s^{CB}. The processing noise η^{CB} is independent of all other random variables and has (infimum) variance

$$\sigma^2_{\eta,CB} = \frac{1-k^{CB}}{k^{CB}} Var(q^*) \tag{29}$$

where $k^{CB} = 1 - 2^{-2K^{CB}} \in [0,1]$ denotes the central bank's processing index. To describe central bank behavior linear policies of the form

$$q = E\left[q^*|s^{CB}\right] = k^{CB}s^{CB} \tag{30}$$

will be considered. Since policy can freely choose the reaction coefficients a and b in equation (28), choosing the reaction coefficient with respect to the signal s^{CB} in equation (30) is without loss of generality.[18]

Firms' behavior remains described by the equations derived in section 5, which allows to express the central bank's maximization problem under com-

[18] The optimality of a linear reaction function follows from the linear quadratic nature of the policy problem, see problem (31) below.

mitment as

$$\max_{a,b} -E[(y-y_n)^2] \tag{31}$$

$$\begin{aligned}
s.t. &: \\
y - y_n &= (1-k)\,q - (1-k)y_n - \frac{k}{\xi}\varepsilon - k\eta \\
\sigma_\eta^2 &= \frac{1-k}{k} Var\,(p^*) \\
p^* &= q - y_n + \frac{1}{\xi}\varepsilon \\
q &= k^{CB}\left(a\varepsilon + by_n + \eta^{CB}\right) \\
\sigma_{\eta,CB}^2 &= \frac{1-k^{CB}}{k^{CB}} Var(a\varepsilon + by_n)
\end{aligned}$$

Substituting the constraints into the objective function and taking first order conditions delivers that optimal policy is given by

$$\begin{aligned}
a &= 0 \\
b &= 1
\end{aligned}$$

This together with equation (28) shows that optimal policy displays certainty equivalence. The implied volatility of the output gap and the price level are

$$\begin{aligned}
E[(y-y_n)^2] &= \frac{k}{\xi^2}\sigma_\varepsilon^2 + (1-k)\,(1-k^{CB})\sigma_{y_n}^2 \\
E\left[p^2\right] &= \frac{k}{\xi^2}\sigma_\varepsilon^2 + k(1-k^{CB})\sigma_{y_n}^2
\end{aligned}$$

Unlike in the case with perfect central bank information, variations in potential output y_n now have an effect on the output gap to the extent these variations are neither observed by the central bank nor the private sector. Moreover, variations in the output gap perceived by firms but not by the central bank now lead to movements in the price level, as firms try to respond to variations in the natural rate of output that the central bank fails to nominally accommodate due to processing limitations.

With discretionary policy, the policy problem is identical to (31), except that the policymaker treats σ_η^2 as independent of policy decisions. Discretionary optimal policy is to set

$$\begin{aligned}
a &= \frac{k}{1-k}\frac{1}{\xi} \\
b &= 1
\end{aligned}$$

and displays again certainty equivalence. The implied variance of the output gap and inflation are

$$\begin{aligned}
E[(y-y_n)^2] &= \frac{\left(1-k(1-k^{CB})\right)}{1-k}\frac{k}{\xi^2}\sigma_\varepsilon^2 + (1-k)(1-k^{CB})\sigma_{y_n}^2 \\
E\left[p^2\right] &= \frac{\left(k^{CB} + (1-k^{CB})(1-k)^2\right)}{(1-k)^2}\frac{k}{\xi^2}\sigma_\varepsilon^2 + k(1-k^{CB})\sigma_{y_n}^2
\end{aligned}$$

The volatility increase due to discretionary policy is thus again particularly pronounced when firms can process information almost perfectly.

6.2 Endogenizing Firms' Capacity Choice

This section shows that with discretionary policy increased focus on price stability continues to imply lower price and output gap volatility, even if firms choose their information processing capacity optimally.

To model firms' choice of processing capacity, a game with the following sequence of events is considered. First, monetary policy is assigned the weight $\omega \geq 0$ on price stabilization in its objective function (26). Second, firms simultaneously choose their processing index k_i ($i \in [0, 1]$) taking as given the choice of other firms. The costs of acquiring capacity k_i are thereby described by a cost function $c(k_i)$ with strictly positive first and second derivatives.[19] Third shocks realize, thereafter the policymaker (discretionarily) determines and implements monetary policy. Fourth, firms process information about shocks and policy decisions and set their prices. Finally production and consumption takes place.

The effects of increased emphasis on price stabilization by the central bank (larger ω) for aggregate volatility are summarized in the following proposition:

Proposition 5 *Suppose firms' capacity choice problem has an interior solution and for each policy weight ω there exists a unique symmetric Nash equilibrium $k^*(\omega)$ for firms' capacity choice. A marginal increase in ω then results in a new symmetric Nash equilibrium with lower output gap and lower price volatility, for all ω sufficiently small.*

As is the case with exogenous processing capacity, increased focus on price stabilization initially lowers aggregate nominal and real volatility. The assumptions stated at the beginning of the proposition thereby simply insure that meaningful comparative statics are associated with a change in the policy weight ω. The proof is provided in Appendix A.2. The crucial step in the proof consists of showing that firms' equilibrium choice for processing capacity is decreasing in the policy weight ω, i.e., that firms choose to observe mark-up shocks less precisely, if the policymaker focuses more on price stability. Reduced processing capacity dampens firms' price response to (their imperfect signal about) mark-up shocks and this contributes further to reducing price and output gap volatility. The reduction in aggregate volatility is thus even larger once one allows firms to optimally choose their information processing capacity.

6.3 Idiosyncratic Processing Errors

This section relaxes the common knowledge assumption, i.e., the assumption that the information processing errors in equation (12) are identical across firms. While it appears reasonable to assume that there is some common component in firms' processing errors, e.g., because firms all read the same (noisy) newspaper reports, it is equally reasonable to postulate the presence of idiosyncratic

[19] If each bit of processing capacity can be acquired at a constant marginal cost, then the cost function expressed in terms of the processing index k satisfies these properties.

processing errors. This section shows that the previous analysis readily extends to a setting with such idiosyncratic information sets.

Suppose that equation (12) is replaced by

$$s^i = p^* + \eta^c + \eta^i$$

where η^c denotes a processing error common to all firms, while η^i is firm i's idiosyncratic processing error that is assumed independent of η^c and p^*. Let a share $\alpha \in [0,1]$ of the total noise be common to all firms with the remaining share being idiosyncratic, i.e.,

$$\begin{aligned}\sigma^2_{\eta^c} &= \alpha\sigma^2_\eta \\ \sigma^2_{\eta^i} &= (1-\alpha)\sigma^2_\eta\end{aligned}$$

where the total observation noise σ^2_η is given by (13). For $\alpha = 1$ one recovers the common knowledge setup studied thus far in the paper, while for $\alpha = 0$ one obtains a setting with purely idiosyncratic noise. The latter has been studied in Adam (2007) where it is shown that a firm's optimal price in the presence of idiosyncratic information noise is given by[20]

$$p(i) = \xi E\left[\sum_{m=0}^{\infty}(1-\xi)^m \left(p^{*(m)}\right)|s^i\right] \tag{32}$$

with $p^{*(m)}$ denoting the so-called m-th order expectation of the optimal price p^*, which can be defined recursively as

$$\begin{aligned}p^{*(0)} &= p^* \\ p^{*(m)} &= \int\limits_{i\in[0,1]} E\left[p^{*(m-1)}|s^i\right] \mathrm{di}\end{aligned}$$

As shown in appendix A.3 the higher-order expectations are given by

$$E^i\left[p^{*(m)}\right] = k\left(k(1-\alpha)+\alpha\right)^m s^i$$

so that equation (32) implies the aggregate price level to be given by

$$\begin{aligned}p &= \int\limits_{i\in[0,1]} p(i)\mathrm{di} \\ &= \frac{\xi k}{1-(1-\xi)\left(k(1-\alpha)+\alpha\right)}\left(q - y_n + \frac{1}{\xi}\varepsilon + \eta^c\right)\end{aligned} \tag{33}$$

For $\alpha = 1$ this expression simplifies to the price level expression derived for the common knowledge setting studied thus far, see for example equation (27). For $\alpha > 0$ it differs from the common knowledge expression only by a proportionality factor and the fact that η^c instead of η enters as the noise term. Since the variance of η^c is proportional to the variance of η, the qualitative behavior

[20] This follows from equation (24) in Adam (2007), which is valid independently of the assumed information structure.

of the aggregate price level does not change when introducing idiosyncratic processing errors. In particular, overly activist discretionary policy will still lead to excessive (real and nominal) aggregate volatility, with the volatility increasing without bound as $k \to 1$.

What would happen if firms could choose whether to observe a common noisy signal or a signal with idiosyncratic noise, i.e., decide about their preferred value for $\alpha \in [0, 1]$? The outcome then depends on the value of the parameter ξ. For $\xi < 1$ firms' prices are strategic complements, i.e., an individual firm wishes to charge a higher (lower) price if other firms charge higher (lower) prices.[21] Strategic complementarity in price setting is sometimes referred to as 'real rigidities', see Ball and Romer (1990), and is the standard assumption in the monetary policy literature. The coordination motive then implies that each firm has an incentive to acquire the same information as other firms resulting in $\alpha = 1$, which is the common knowledge setting analyzed in the baseline setup.

7 Infinite Horizon Economy

This section extends the static model to an infinite horizon economy with persistent shocks, showing how the previous findings naturally extend to a fully intertemporal setup. It demonstrates that changes in the conduct of monetary policy induce changes in the model-implied time series that are consistent with a number of stylized facts on the Great Moderation documented in the empirical literature. This literature appears to largely agree on the following set of findings:

Finding 1: Aggregate real and nominal volatility pre 1984 was significantly higher than thereafter. This is the case for the United States but also for a number of other industrialized economies (McConnell and Quiros (2000), Blanchard and Simon (2001)).

Finding 2: VAR evidence for the pre and post 1984 periods indicates that the different levels of volatility are mainly the result of larger VAR residuals and only to a lesser extent the result of a change in the autoregressive coefficients of the VAR (Sims and Zha (2006), Primiceri (2005)).

Finding 3: When exporting the monetary policy equation of a VAR estimated from the post 1984 period to the pre 1984 period (or vice versa) while keeping unchanged the remaining equations of the estimated VAR as well as the VAR shocks, then this leads to virtually unchanged economic outcomes (Canova and Gambetti (2008), Sims and Zha (2006), Primiceri (2005)).

Finding 4: It appears that only a negligible share of the variance reduction following 1984 is due to reduced variance of monetary policy shocks (Sims and Zha (2006)).

It should be noted that there is some disagreement in the literature about 'Finding 2'. Cogley and Sargent (2005), for example, argue that changes in the autoregressive coefficients are also relevant but that these are statistically harder to detect. Also, Boivin and Giannoni (2006) argue that the impulse response to monetary policy shocks has changed, which equally points to changes in

[21] See Adam (2007) for details.

the autoregressive coefficients of the reduced form dynamics. Despite these alternative findings, I proceed by treating 'Finding 2' as a well-established fact, simply because these findings are sometimes informally interpreted as ruling out the possibility that the observed volatility changes are due to changes in the systematic conduct of monetary policy. The focus is thus on empirical findings that - at first sight - appear to be most biased against a monetary policy based explanation of the 'Great Moderation'.

The next section introduces the dynamic model and derives optimal policy with and without a conservative objective function. Section 7.2 demonstrates that all four findings listed above can be generated by installing a conservative central banker.

7.1 Dynamic Model

This section describes the infinite horizon economy and determines the optimal monetary policy. As in most of the earlier setup, it abstracts from information processing errors by the monetary authority, so that monetary policy itself is not a source of randomness in the economy. This implies that monetary policy shocks are absent form the outset, so that the model is consistent with Finding 4 mentioned in the previous section.

Infinite Horizon Economy Consider an infinite horizon economy without capital in which the firm side is identical to the one described in section 3. The representative consumer now maximizes

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(U(Y_t) - \nu_t V(L_t) + \chi_t D(M_t/P_t) \right) \right] \tag{34}$$

$$s.t.$$

$$P_t Y_t + B_t + M_t = W_t L_t + \Pi_t - T_t + R_{t-1} B_{t-1} + M_{t-1}$$

where M_t denotes nominal money balances, B_t nominal bonds, R_t the nominal interest rate, and χ_t a money demand shock with $E[\chi_t] = \chi$. Previously used variables retain their definition from section 3. To simplify the analysis, a 'cashless' limit economy ($\chi \to 0$) will be considered. This allows to abstract from the utility implications of monetary policy actions that operate through level of real cash balances.

The government flow budget constraints is

$$B_t + M_t = M_{t-1} + B_{t-1} R_{t-1} - T_t + \tau Y_t$$

The government chooses an efficient output subsidy τ to eliminate the steady state distortions from monopolistic competition. In addition, it chooses a sequence of conditional debt and lump sum tax plans (B_t, T_t), which are assumed to give rise to a bounded path for the real value of outstanding government claims. The latter implies that Ricardian equivalence holds so that the fiscal choices for (B_t, T_t) do not affect the equilibrium outcome. One can thus abstract from fiscal policy when analyzing the conduct of monetary policy.

LQ-Approximation of the Monetary Policy Problem I now derive a linear-quadratic approximation to the monetary policy problem for the dynamic economy. As in the static model, monetary policy maximizes social welfare, where the policy choices are subject to a number of implementability constraints. The constraints consist of firms' price setting equation and the laws governing firms' beliefs under optimal information processing. As before, the monetary policy 'instrument' is assumed to be nominal demand rather than the nominal interest rate or some monetary aggregate. This is motivated by analytical convenience and to increase comparability with the findings from the static model. The equilibrium outcomes remain unaffected by this assumption.[22]

A quadratic approximation of the utility of the representative household (34) is given by

$$-E_0[\sum_{t=0}^{\infty} \beta^t (y_t - y_{n,t})^2]$$

where $y_{n,t}$ denotes again the efficient output level.[23] For simplicity, consider the limiting case $\beta \to 1$, which allows to express household utility and thus the monetary policy objective as

$$-E\left[(y_t - y_{n,t})^2\right] \tag{35}$$

where $E[\cdot]$ denotes the unconditional expectations operator.

For a given information set, the linear approximation to profit maximizing price setting behavior by firms continues to be described by equations (8) and (9), i.e.:

$$p_t = E\left[q_t - y_{n,t} + \frac{1}{\xi}\varepsilon_t | I_t\right] \tag{36}$$

where I_t denotes firms' information set at time t. The stochastic disturbances $x_t = (\varepsilon_t, y_{n,t})'$ thereby evolve according to[24]

$$x_t = \rho x_{t-1} + v_t \tag{37}$$

where $\rho \in (-1, 1)$ and $v_t \sim iiN(0, \Sigma_v)$ with

$$\Sigma_v = \begin{pmatrix} \sigma_{v_1}^2 & 0 \\ 0 & \sigma_{v_2}^2 \end{pmatrix}.$$

At the time when monetary policy is implemented, the economy is characterized by the state variables x_t and $x_{t|t-1}$ where the latter denote firms' $t-1$ expectations of x_t, see the timeline in figure 1.

In general, optimal policy should condition its choices on x_t as well as on $x_{t|t-1}$. Yet, nominal demand variations that depend on private sector beliefs

[22] This is so because the paper effectively considers Ramsey allocation problems with and without commitment and does not address issues of equilibrium implementation. As can be easily shown, the equilibrium path for prices, output gap and nominal demand derived below imply a corresponding path for the nominal interest rate and money demand. The choice of policy instrument can matter, however, for implementing a desirable allocation as the unique equilibrium outcome. This issue is beyond the scope of this paper.

[23] See equation (56) in appendix A.1 for the definition of y_n.

[24] At the cost of additional notational complexity one could easily allow for shocks with different persistence.

are fully perceived by firms, therefore do not generate real effects and can be safely ignored.[25] This allows considering policy of the form

$$q = a \cdot \varepsilon_t + b \cdot y_{n,t} \tag{38}$$

as in the static setup. Given policy of this form, firms' optimal signal (12) can be written as

$$s_t = H' x_t + \eta_t \tag{39}$$

where

$$H' = (\frac{1}{\xi} + a, b - 1)$$

The state equations (37) and the observation equation (39) together define firms' Kalman filtering problem. Letting $x_{t|t}$ denote agents' time t estimate of x_t, appendix A.5 shows that the Kalman filter updating equations imply the following evolution of firms' beliefs under optimal information processing:

$$\begin{aligned} H' x_{t|t} &= (1-k) H' x_{t|t-1} + k s_t \\ &= \rho(1-k) H' x_{t-1|t-1} + k s_t \end{aligned} \tag{40}$$

This filtering problem has two non-standard features. First, the observation equation H depends on the policy parameters a and b. This is due to firms being able to *choose* which variables to observe through the information channel. Firms' choice thereby depends on the policy pursued by the central bank. Second, the variance of the observation noise η_t in equation (39) is endogenous since the information flow generated by the signal s_t is constrained by firms' information processing capacity.[26]

Defining the state of the economy as $\Xi_t = \left(x_t', H' x_{t|t}\right)$ and using the previous results, one can write the evolution of the state as

$$\Xi_t = A \cdot \Xi_{t-1} + B \omega_t \tag{41}$$

where the state innovation vector

$$\omega_t' = (v_{1t}, v_{2t}, \eta_t)' \tag{42}$$

consists of the innovations to fundamentals (price mark-up, natural output level) *and of innovations to firms information sets (η_t) that are the result of processing errors.*[27] The latter will be crucial for interpreting the findings that follow. Explicit expressions for the matrices A and B are given in appendix A.4.

The equilibrium price, the output gap, and monetary policy are functions of the state vector

$$\begin{pmatrix} p_t \\ y_t - y_{n,t} \\ q_t \end{pmatrix} = C \cdot \Xi_t \tag{43}$$

where the explicit expression for C can be found in appendix A.4.

[25] This would be different in a setting with idiosyncratic information processing errors.

[26] In particular, $VAR(\eta_t) = \frac{1-k}{k} H' P_{t|t-1} H$ where $P_{t|t-1}$ denotes agents' uncertainty about x_t prior to observing s_t.

[27] For the moment, I abstract from the money demand shocks χ_t in the state vector, as these do not affect the equilibrium dynamics of the variables under consideration.

The linear quadratic policy problem now consists of choosing the policy reaction coefficients (a, b) so as to maximize objective (35) subject to equations (41) and (43), which summarize optimal price setting behavior and optimal information processing by firms. Under commitment, the policymaker thereby recognizes that the variance of the observation error η_t entering the state innovation vector ω_t in equation (41) is a function of its policy. Under discretionary policy, the policymaker treats the variance of observation errors as given, as was the case in the static model.

Optimal Policy and Monetary Conservatism The following proposition shows that the results from the static model carry over in a natural way to the infinite horizon setting. In particular, discretionary monetary policy suboptimally accommodates mark-up shocks with the degree of accommodation increasing in firms' processing capacity and becoming unbounded in the limit as firms process information perfectly. Moreover, a conservative and discretionary central bank will accommodate less and implement the commitment solution for an appropriate degree of conservatism.

Proposition 6 *Optimal discretionary policy is given by*

$$a = \frac{k}{\xi(1-k)(1-\rho^2(1-k))} > 0$$
$$b = 1$$

while under commitment it is optimal to choose

$$a = 0$$
$$b = 1$$

A conservative and discretionary monetary policymaker maximizing

$$-E\left[(y_t - y_{n,t})^2 + \omega p_t^2\right]$$

with $\omega \geq 0$ *will choose* $a(\omega)$ *and* $b = 1$ *with* $\partial a(\omega)/\partial \omega < 0$ *and* $\lim_{\omega \to \infty} a(\omega) = -\frac{1}{\xi}$.

The proof is given in appendix A.5.

7.2 Model Implied VAR Dynamics

This section shows that the dynamics of prices, output gap, and monetary policy follow a first-order vector-autoregression (VAR). Moreover, following the installment of a conservative central banker, the changes in the VAR dynamics are shown to reproduce Findings 1-3 mentioned at the beginning of section 7.[28]

Throughout this section, monetary policy in the VAR is identified with nominal demand to stay consistent with the previous analysis. Section 7.3 shows that the main conclusions extend to a VAR involving interest rates instead of

[28] As discussed at the beginning of section 7.1, Finding 4 is also replicated because the model abstracts from monetary policy shocks.

nominal demand and to an augmented VAR involving also a monetary aggregate.

The following proposition summarizes the model implications for the dynamics of prices, output gap and monetary policy:

Proposition 7 *Suppose $a \neq 0$ and let $z_t' = (p_t, y_t - y_{n,t}, q_t)'$. Then*

$$z_t = Dz_{t-1} + u_t \tag{44}$$

with

$$D = \begin{pmatrix} \frac{(bk+a\xi)\rho}{\xi a} & \frac{(b+a\xi)k\rho}{\xi a} & \frac{(1-b)k\rho}{\xi a} \\ -\frac{\rho kb}{\xi a} & -\frac{(bk-a\xi+ak\xi)\rho}{\xi a} & \frac{(b-1)k\rho}{\xi a} \\ 0 & 0 & \rho \end{pmatrix}$$

and

$$u_t = CB\omega_t$$

with $\omega_t \in R^3$ denoting the vector of fundamental shocks defined in (42).

Proof. If $a \neq 0$, one can invert the matrix C in equation (43) and use it to substitute Ξ_t and Ξ_{t-1} in (41), which delivers the stated result. ■

Importantly, the variables z_t entering the VAR in equation (44) fully reveal the state vector Ξ_t, provided $a \neq 0$. This implies that the subsequent results cannot be overturned by adding additional information (observables) to the VAR: observing the variables in z_t is already the best situation an econometrician can hope for.

The expression for the autoregressive matrix D in proposition 7 shows that a change in the monetary policy reaction coefficients (a, b) does not affect the last row of this matrix. Therefore, monetary policy affects only the impact matrix CB pre-multiplying the structural economic shocks and the rows in the autoregressive matrix D governing the evolution of prices and output-gaps. Furthermore, if $b = 1$ as is the case with discretionary policy (with or without conservatism) as well as with optimal commitment policy, the reduced form dynamics for prices and output gap in the VAR do not depend on lagged nominal demand! This is so because lagged nominal demand depends on lagged natural rate shocks, while the output gap and the price level do not.[29] This in turn implies that if a researcher estimates VARs from two policy regimes with different degrees of nominal accommodation of mark-up shocks and exchanges the identified monetary policy reaction function in the VAR across the two regimes to run counter-factual policy simulations (while leaving untouched the structural shocks hitting each regime), the researcher will find that such a change makes no difference for the dynamics of prices and output gap. This is the case, even though all actual changes in the model are triggered by a change in monetary policy. This shows that the model is consistent with Finding 3 mentioned in the previous section.

[29] At this point it is crucial that there exists a shock that does not generate a trade-off between output gap and price stabilization.

Let us now turn consideration to the empirical evidence mentioned in Findings 1 and 2 above. The following preliminary result establishes that the variance of all model implied VAR residuals falls after installing a conservative central bank. The proof is in appendix A.6.

Lemma 8 *Suppose $a > 0$ and $b = 1$. The variance of all VAR residuals is strictly increasing in a.*

The next lemma establishes that the appointment of a conservative central banker gives rise to Finding 1. The fact that variance of prices, output gap and the monetary policy instrument fall does not directly follow from the previous lemma because changes in a can also affect the autoregressive matrix D in equation (44), with some of the autoregressive coefficients possibly strongly increasing as a falls to levels close to zero.

Lemma 9 *Suppose $a > 0$ and $b = 1$. The unconditional variance of prices, output gap, and monetary policy are strictly increasing in a.*

The proof can be found in appendix A.7. As in the static model, the variance reduction is due to the 'coordination effect'. The next result shows that arbitrarily large variance reductions can result from installing a conservative central bank:

Lemma 10 *Consider a regime shift from discretionary optimal policy towards fully optimal policy, see proposition 6. This generates an arbitrarily large (relative and absolute) reduction in the volatility of prices and the output gap, provided firms can process information sufficiently well (k sufficiently close to 1).*

The proof is given in appendix A.8. As in the static case, the volatility reduction can be arbitrarily large because the volatility generated by discretionary policy increases without bound as firms start to process information almost perfectly ($k \to 1$).

Finally, the subsequent lemma shows that, consistent with Finding 2 mentioned at the beginning of section 7, the model is consistent with the notion that monetary conservatism operates mainly through the VAR residuals and only to a lesser extend through changes in the VAR's autoregressive coefficients, provided firms are sufficiently good at processing information. The proof is found in A.9.

Lemma 11 *Evaluating the following derivatives at the discretionary policy solution from proposition 6 delivers*

$$\lim_{k\to 1} \frac{\partial D}{\partial a} = 0_{3x3} \tag{45}$$

$$\lim_{k\to 1} \frac{\partial diag(VAR(u_t))}{\partial a} \geq \frac{2\sigma_{v_1}^2}{\xi} \cdot 1_{3x1} \tag{46}$$

This establishes that following the installation of a conservative central banker, the model can reproduce Findings 1-4 listed at the beginning of section 7.

7.3 VAR with Interest Rates and Monetary Aggregates

The previous section identified monetary policy in the VAR using nominal demand. Clearly, this is at odds with much of the empirical literature that - following the monetary policy practice of recent decades - tends to use nominal interest rates instead. This section shows how the previous extend to a setting where nominal interest rates are used as monetary policy instrument in the VAR. In addition, it illustrates that all previous results remain unaffected when augmenting the VAR with a monetary aggregate.

Linearizing the consumption Euler equation implied by the first order conditions of (34) delivers

$$\begin{aligned} i_t &= -\left(\frac{U''\overline{Y}}{U'}\right)(E_t y_{t+1} - y_t) + E_t p_{t+1} - p_t \\ &= (E_t q_{t+1} - q_t) - \left(\frac{U''\overline{Y}}{U'} + 1\right)(E_t y_{t+1} - y_t) \end{aligned} \tag{47}$$

Assuming log utility in consumption, the short-term interest rate consistent with the path for nominal demand implied by equation (44) is given by

$$i_t = (\rho - 1)q_t$$

With log consumption utility all previous results, therefore, fully extend to a VAR including short-term nominal interest rates instead of nominal demand. If consumption utility deviates from the log case, then current and expected future output affect nominal interest rates, see equation (47). Since the evolution of output depends on the policy coefficients a and b, see equation (44), the autoregressive coefficients of the VAR for the interest rate equation will generally not be independent of policy anymore. This may potentially cause a policy shift to show up in the autoregressive coefficients. This could cause the model to be at odds with Finding 3 mentioned in section 7. To what extend the AR coefficients of the interest rate equation do indeed change depends on the degree to which consumption utility deviates from log utility. It also appears that the empirical literature is not at odds with the notion that these coefficients have indeed changed somewhat, although not by a statistically significant amount.

Finally, consider the effects of including monetary aggregates into the VAR. The first order conditions of problem (34) deliver a money demand equation of the form

$$D'_t = \frac{1}{\chi_t}\frac{(R_t - 1)}{R_t}U'_t$$

Linearizing this equation and using (47) to substitute nominal interest rates, delivers a linearized money demand equation of the form

$$m_t = \alpha_0 q_t + \alpha_1 p_t + \alpha_2 \xi_t \tag{48}$$

where m_t denotes nominal balances[30], $\xi_t = (\chi_t - \chi)/\chi$ money demand shocks, and α_i linearization coefficients ($i = 0, 1, 2$). One can can easily augment the

[30] Expressed in percent deviation form its deterministic level.

VAR in equation (44) with the evolution of monetary aggregates implied by equation (48). Since the dynamics of output gap, prices, and policy instrument remain unaffected, all previously derived results carry over to such an augmented VAR.

8 Conclusions

Taking into account the endogeneity of decision makers' information structures appears to have important implications for the conduct of monetary policy and stabilization policy more generally. Stabilization policies become counterproductive if the achievement of the stabilization goals causes optimal private sector decisions to become very volatile. Volatility of private sector decisions considerably complicates the information processing problems faced by private agents, so that their decisions become contaminated by large and unpredictable noise components which may end up increasing aggregate volatility. In empirical applications these processing errors would show up as a fundamental source of randomness, although their variance ultimately depends on the conduct of stabilization policy.

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A Web Appendix (not for publication)

A.1 Price setting equation and welfare objective

In a first step the linearized price setting equation (6) is derived. The product demand functions associated with the Dixit-Stiglitz aggregator (2) are

$$Y^i(P^i) = \left(P^i/P\right)^{-\theta} Y \tag{49}$$

where

$$P = \left(\int \left(P^j\right)^{1-\theta} dj\right)^{\frac{1}{1-\theta}} \tag{50}$$

Using (49) the first order condition of the firms' profit maximization problem (3) delivers

$$P^i = E\left[\frac{1}{1+\tau}\frac{\theta}{\theta-1}W|I\right] \tag{51}$$

In a symmetric equilibrium $P^i = P$. Equation (49) then implies $Y^j = Y$ and the household's first order condition can be written as

$$W = \frac{\nu V'(Y)}{U'(Y)}P \tag{52}$$

Combining (51) and (52) delivers

$$P^i = E\left[\frac{1}{1+\tau}\frac{\theta}{\theta-1}\frac{\nu V'(Y)}{U'(Y)}P|I\right] \tag{53}$$

In the symmetric deterministic steady state $P^i = P = \overline{P}$, $Y^i = Y = \overline{Y}$, $\theta = \overline{\theta}$, and $\nu = 1$ where $\overline{Y}$ solves

$$\frac{1}{1+\tau}\frac{\overline{\theta}}{\overline{\theta}-1}\frac{V'(\overline{Y})}{U'(\overline{Y})} = 1 \tag{54}$$

and $\overline{P}$ is any value chosen by the central bank. Steady state output $\overline{Y}$ is first best for

$$\tau = \frac{1}{\overline{\theta}-1}$$

which implies

$$V'(\overline{Y}) = U'(\overline{Y}) \tag{55}$$

For a given the labor supply shock ν, the first best output level Y_n solves

$$\frac{\nu V'(Y_n)}{U'(Y_n)} = 1$$

Linearizing this equation around the steady state delivers

$$\nu - 1 = -\frac{V''(\overline{Y})U'(\overline{Y}) - V'(\overline{Y})U''(\overline{Y})}{\left(U'(\overline{Y})\right)^2}\overline{Y}\left(\frac{Y_n - \overline{Y}}{\overline{Y}}\right) \tag{56}$$

Linearizing (53) around the deterministic steady state and using (56) delivers (6) where:

$$\begin{aligned}
\varepsilon_t &= -\frac{1}{\overline{\theta}-1}\frac{(\theta-\overline{\theta})}{\overline{\theta}} \\
\xi &= \frac{V''(\overline{Y})U'(\overline{Y}) - V'(\overline{Y})U''(\overline{Y})}{\left(U'(\overline{Y})\right)^2}\overline{Y} \\
&= \frac{V''(\overline{Y})\overline{Y}}{V'(\overline{Y})} - \frac{U''(\overline{Y})\overline{Y}}{U'(\overline{Y})}
\end{aligned}$$

The second step derives the welfare approximation (4). Consider a symmetric equilibrium where $P^i = P$ and $Y^i = Y$. A second order approximation of the utility Ω of the representative agent around the steady state level $\overline{\Omega}$ is then given by

$$\begin{aligned}
\Omega - \overline{\Omega} = &\, U'(\overline{Y})(Y-\overline{Y}) - V'(\overline{Y})(Y-\overline{Y}) \\
&+ \frac{1}{2}U''(\overline{Y})(Y-\overline{Y})^2 - \frac{1}{2}V''(\overline{Y})(Y-\overline{Y})^2 \\
&- V'(\overline{Y})(Y-\overline{Y})(\nu-1) + O(2) + t.i.p \qquad (57)
\end{aligned}$$

where $t.i.p.$ denotes (first and higher order) terms that are independent of policy and $O(2)$ summarizes endogenous terms of order larger than two. Using equations (55) and (56) and adding $\frac{1}{2}\left[U''(\overline{Y}) - V''(\overline{Y})\right]\left(Y^n - \overline{Y}\right)^2$, which is a term independent of policy, the welfare approximation (57) can be written as

$$\begin{aligned}
\Omega - \overline{\Omega} = &\, \frac{1}{2}\left[U''(\overline{Y}) - V''(\overline{Y})\right]\overline{Y}^2\left(\frac{(Y-\overline{Y})}{\overline{Y}} - \frac{(Y^n-\overline{Y})}{\overline{Y}}\right)^2 \\
&+ O(2) + t.i.p \qquad (58)
\end{aligned}$$

which is of the form postulated in (4).

A.2 Proof of proposition 5

Firm i chooses its capacity index k_i so as to

$$\max_{k_i} \Pi(k_i, k_{-i}, \omega) - c(k_i)$$

where $\Pi(k_i, k_{-i}, \omega)$ denotes the firm's expected profits from selling goods and $c(k_i)$ the costs of acquiring processing capacity. The firm takes as given the policy weight ω placed on price stabilization as well as the capacity choice k_{-i} of other firms. Under the maintained assumptions, the firm's optimal capacity choice k_i^* solves

$$\Pi_{k_i}(k_i^*, k_{-i}, \omega) - c_{k_i}(k_i) = 0 \qquad (59)$$

and there is a unique symmetric Nash equilibrium k^* solving

$$\Pi_{k_i}(k^*, k^*, \omega) - c_{k_i}(k^*) = 0 \qquad (60)$$

The price and output gap variances implied by an (arbitrary) capacity choice k and a policy weight ω are

$$
\begin{aligned}
Var(p) &= \frac{(1-k)^2 k}{\left((1-k)^2 + \omega k^2\right)^2 \xi^2}\sigma_\varepsilon^2 \\
Var\left(y - y_n\right) &= \frac{\left(3k^2 - 3k - k^3 + k^3\omega^2 + 1\right)}{\left((1-k)^2 + \omega k^2\right)^2}\frac{k}{\xi^2}\sigma_\varepsilon^2
\end{aligned}
$$

The marginal effects of ω on the Nash equilibrium are thus

$$
\begin{aligned}
\frac{dVar(p)}{d\omega} &= \frac{\partial Var(p)}{\partial \omega} + \frac{\partial Var(p)}{\partial k}\frac{\partial k^*}{\partial \omega} \\
\frac{dVar(y-y_n)}{d\omega} &= \frac{\partial Var(y-y_n)}{\partial \omega} + \frac{\partial Var(y-y_n)}{\partial k}\frac{\partial k^*}{\partial \omega}
\end{aligned}
$$

As is easily verified $\partial Var(p)/\partial\omega < 0$, $\partial Var(y - y_n)/\partial\omega < 0$, $\partial Var(p)/\partial k > 0$, and $\partial Var(y - y_n)/\partial k > 0$ for ω sufficiently small. One thus has $dVar(p)/d\omega < 0$ and $dVar(y - y_n)/d\omega < 0$, provided $\frac{\partial k^*}{\partial \omega} < 0$. From the implicit function theorem and (60)

$$
\begin{aligned}
\frac{\partial k^*}{\partial \omega} &= -\frac{\partial\left(\Pi_{k_i}(k^*, k^*, \omega) - c_{k_i}(k^*)\right)/\partial\omega}{\partial\left(\Pi_{k_i}(k^*, k^*, \omega) - c_{k_i}(k^*)\right)/\partial k_i} \\
&= -\frac{\Pi_{k_i\omega}(k^*, k^*, \omega)}{\Pi_{k_ik_i}(k^*, k^*, \omega) - c_{k_ik_i}}
\end{aligned}
$$

We know $c_{k_ik_i} > 0$ and below we show that $\Pi_{k_ik_i}(k^*, k^*, \omega) = 0$ and $\Pi_{k_i\omega}(k^*, k^*, \omega) < 0$. This together establishes $\frac{\partial k^*}{\partial \omega} < 0$. The sign of the derivatives $\Pi_{k_ik_i}$ and $\Pi_{k_i\omega}$ can be determined using the second order approximation to firms' profits (10):

$$
\begin{aligned}
E\left[-E\left[\left(p^i - p^*\right)^2 | I\right]\right] &= -E\left[\left(k_i s^i - p^*\right)^2\right] \\
&= -E\left[\left(k_i p^* + k_i \eta - p^*\right)^2\right] \\
&= -(1-k_i)^2 E\left[\left(p^*\right)^2\right] - \left(k_i\right)^2 E\left[\left(\eta^i\right)^2\right]
\end{aligned}
$$

Optimal discretionary policy for given ω and k is

$$
q = \frac{\left((1-k)k - \omega k^2\right)}{\left((1-k)^2 + \omega k^2\right)}\frac{1}{\xi}\varepsilon + y_n
$$

and implies

$$
E[(p^*)^2] = \left(\frac{(1-k)}{\left((1-k)^2 + \omega k^2\right)}\frac{1}{\xi}\right)^2 \sigma_\varepsilon^2
$$

and

$$
E\left[\left(\eta^i\right)^2\right] = \frac{1-k_i}{k_i}E[(p^*)^2]
$$

The quadratic approximation of firms' profits is thus

$$E\left[-E\left[\left(p^i-p^*\right)^2|s^i\right]\right]=\frac{\left(k_i-1\right)\left(k-1\right)^2}{\left(\left(1-k\right)^2+k^2\omega\right)^2\xi^2}\sigma_\varepsilon^2$$

and implies

$$\begin{aligned}\Pi_{k_ik_i}(k^*,k^*,\omega)&=0\\ \Pi_{k_i\omega}(k^*,k^*,\omega)&=-2\frac{\left(1-k^*\right)^2\left(k^*\right)^2}{\left(\left(1-k^*\right)^2+\left(k^*\right)^2\omega\right)^3\xi^2}\sigma_\varepsilon^2<0\end{aligned}$$

which establishes the claim.■

A.3 Higher-Order Expectations

We know that

$$\begin{aligned}\sigma_\eta^2&=\frac{1-k}{k}Var(p^*)\\ var(s^i)&=\frac{1}{k}Var(p^*)\\ cov(p^*,s^i)&=var(p^*)\\ cov(p^*+\eta^c,s^i)&=\frac{\alpha(1-k)+k}{k}Var(p^*)\end{aligned}$$

Using the updating formulae for the conditional mean of jointly normally distributed random variables one gets

$$E\left[p^*|s^i\right]=ks^i$$

and

$$p^{*(1)}=\int\limits_{i\in[0,1]}E\left[p^*|s^i\right]=k\left(p^*+\eta^c\right)$$

Furthermore,

$$\begin{aligned}E\left[p^{*(1)}|s^i\right]&=kE\left[\left(p^*+\eta^c\right)|s^i\right]\\ &=k\left(\frac{cov(p^*+\eta^c,s^i)}{var(s^i)}s^i\right)\\ &=k\left(k(1-\alpha)+\alpha\right)s^i\end{aligned}$$

so that

$$p^{*(2)}=k\left(k(1-\alpha)+\alpha\right)\left(p^*+\eta^c\right)$$

and

$$\begin{aligned}E\left[p^{*(2)}|s^i\right]&=k\left(k(1-\alpha)+\alpha\right)E\left[\left(p^*+\eta^c\right)|s^i\right]\\ &=k\left(k(1-\alpha)+\alpha\right)^2s^i\end{aligned}$$

Repeatedly integrating over i and taking the expectations $E\left[\cdot|s^i\right]$ delivers:

$$E\left[p^{*(m)}|s^i\right]=k\left(k(1-\alpha)+\alpha\right)^m s^i$$

A.4 Details on the State Dynamics

The matrices in equation (41) are defined as follows:

$$A = \begin{pmatrix} \rho & 0 & 0 \\ 0 & \rho & 0 \\ k\rho\left(a+\frac{1}{\xi}\right) & k\rho\,(b-1) & \rho(1-k) \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ k\left(a+\frac{1}{\xi}\right) & k\,(b-1) & k \end{pmatrix}$$

The matrix in equation (43) is given by

$$C = \begin{pmatrix} 0 & 0 & 1 \\ a & (b-1) & -1 \\ a & b & 0 \end{pmatrix}$$

A.5 Optimal Policy in the Dynamic Economy

I start by deriving the commitment solution. Using (43) the policy objective can be expressed as

$$\begin{aligned} -E\left[(y-y_{n,t})^2\right] &= -E\left[\left(a\varepsilon_t + (b-1)y_{n,t} - H'x_{t|t}\right)^2\right] \\ &= -\left[a^2\sigma_\varepsilon^2 + (b-1)^2\sigma_y^2 + E\left[(H'x_{t|t})^2\right]\right. \\ &\quad \left. -2a\cdot cov(\varepsilon_t, H'x_{t|t}) - 2(b-1)\cdot cov(y_{n,t}, H'x_{t|t})\right] \end{aligned} \tag{61}$$

I first derive an explicit expression for $H'x_{t|t}$ that allows to compute the variance and the covariance terms appearing in (61). The Kalman filter updating equations are

$$x_{t|t} = x_{t|t-1} + P_{t|t-1}H(H'P_{t|t-1}H + \sigma_\eta^2)^{-1}(s_t - H'x_{t|t-1}) \tag{62}$$

$$P_{t|t} = P_{t|t-1} - P_{t|t-1}H(H'P_{t|t-1}H + \sigma_\eta^2)^{-1}H'P_{t|t-1} \tag{63}$$

where $x_{t|t}$ is the posterior mean of x_t and $x_{t|t-1}$ the prior mean. Likewise, $P_{t|t}$ denotes the posterior covariance matrix of x_t and $P_{t|t-1}$ the prior covariance matrix. Equation (13) implies that the variance of the channel noise is given by

$$\sigma_\eta^2 = \frac{1-k}{k}H'P_{t|t-1}H \tag{64}$$

From equations (63) and (64)

$$\begin{aligned} H'P_{t|t}H &= (1-k)H'P_{t|t-1}H \\ &= (1-k)H'\left(\rho^2 P_{t-1|t-1} + \Sigma_v\right)H \end{aligned} \tag{65}$$

Equation (65) implies that in steady state

$$P = \frac{1-k}{1-(1-k)\rho^2}\Sigma_v \tag{66}$$

Using equations (41) and (64) equation (62) implies

$$\begin{aligned} H'x_{t|t} &= (1-k)H'x_{t|t-1} + ks_t \\ &= \rho(1-k)H'x_{t-1|t-1} + ks_t \end{aligned}$$

and since $|\rho(1-k)| < 1$

$$H'x_{t|t} = \sum_{j=0}^{\infty} ((1-k)\rho)^j \, k(H'x_{t-j} + \eta_{t-j}) \tag{67}$$

Equation (67) implies that

$$cov(\varepsilon_t, H'x_{t|t}) = \frac{k}{1-(1-k)\rho^2} \left(a + \frac{1}{\xi}\right) \sigma_\varepsilon^2 \tag{68}$$

$$cov(y_{n,t}, H'x_{t|t}) = \frac{k}{1-(1-k)\rho^2} \, (b-1) \, \sigma_y^2 \tag{69}$$

Equation (67) also implies that

$$var(H'x_{t|t}) = k^2 \left(E\left[\left(\sum_{j=0}^{\infty} ((1-k)\rho)^j \, H'x_{t-j} \right)^2 \right] + \left[\frac{1}{1-((1-k)\rho)^2} \sigma_\eta^2 \right] \right) \tag{70}$$

Some tedious but straightforward calculations show that

$$E\left[\left(\sum_{j=0}^{\infty} ((1-k)\rho)^j \, H'x_{t-j} \right)^2 \right] = \frac{1}{\left(1-((1-k)\rho)^2\right)} \left(\frac{2}{(1-(1-k)\rho^2)} - 1 \right) H'Var(x_t)H$$

Furthermore, from equation (64)

$$\begin{aligned} \frac{1}{1-((1-k)\rho)^2}\sigma_\eta^2 &= \frac{1}{1-((1-k)\rho)^2} \frac{1-k}{k} H'P_{t|t-1}H \\ &= \frac{1}{1-((1-k)\rho)^2} \frac{1-k}{k} H'(\rho^2 P_{t-1|t-1} + \Sigma_v)H \end{aligned} \tag{71}$$

Using the steady state expression (66) the steady state version of equation (71) is

$$\begin{aligned} \frac{1}{1-((1-k)\rho)^2}\sigma_\eta^2 &= \frac{1}{1-((1-k)\rho)^2} \frac{1}{1-(1-k)\rho^2} \frac{1-k}{k} H'\Sigma_v H \\ &= \frac{(1-\rho^2)}{1-((1-k)\rho)^2} \frac{1}{1-(1-k)\rho^2} \frac{1-k}{k} H'Var(x_t)H \end{aligned} \tag{72}$$

Combining equations (70), (71), and (72) and simplifying delivers

$$E\left[(H'x_{t|t})^2\right] = \frac{k}{1-(1-k)\rho^2} H'Var(x_t)H \tag{73}$$

Substituting (68), (69), and (73) into (61) delivers

$$\begin{aligned}
-E\left[(y-y_{n,t})^2\right] &= -a^2\sigma_\varepsilon^2 - (b-1)^2\sigma_y^2 \\
&- \frac{k}{1-(1-k)\rho^2}\left(\left(a+\frac{1}{\xi}\right)^2\sigma_\varepsilon^2 + (b-1)^2\sigma_y^2\right) \\
&+ 2a\frac{k}{1-(1-k)\rho^2}\left(a+\frac{1}{\xi}\right)\sigma_\varepsilon^2 \\
&+ 2\frac{k}{1-(1-k)\rho^2}(b-1)^2\sigma_y^2 \qquad (74)
\end{aligned}$$

The first order conditions for maximizing (74) with respect to a and b deliver result in the proposition for the commitment case.

Under discretion, the policymaker takes the observation noise σ_η^2 in equation (70) as exogenous. Under discretion the policy objective (74) thus has to be modified to:[31]

$$\begin{aligned}
-E\left[(y-y_{n,t})^2\right] &= -a^2\sigma_\varepsilon^2 - (b-1)^2\sigma_y^2 \\
&- \frac{k^2}{\left(1-((1-k)\rho)^2\right)}\left(\frac{2}{(1-(1-k)\rho^2)}-1\right)\left(\left(a+\frac{1}{\xi}\right)^2\sigma_\varepsilon^2 + (b-1)^2\sigma_y^2\right) \\
&- k^2\left[\frac{1}{1-((1-k)\rho)^2}\sigma_\eta^2\right] \\
&+ 2a\frac{k}{1-(1-k)\rho^2}\left(a+\frac{1}{\xi}\right)\sigma_\varepsilon^2 \\
&+ 2\frac{k}{1-(1-k)\rho^2}(b-1)^2\sigma_y^2 \qquad (75)
\end{aligned}$$

The first order conditions with respect to a and b deliver the result stated in the proposition for the case without commitment.

Next, consider conservative discretionary monetary policy, which maximizes

$$-E\left[(y_t-y_{n,t})^2 + \omega p_t^2\right]$$

Since $p_t = H'x_{t|t}$, equation (73) shows that

$$\begin{aligned}
E_t[p_t^2] &= \frac{k}{1-(1-k)\rho^2}H'Var(x_t)H \\
&= \frac{k}{1-(1-k)\rho^2}\left(\left(a+\frac{1}{\xi}\right)^2\sigma_\varepsilon^2 + (b-1)^2\sigma_{y_n}^2\right)
\end{aligned}$$

so that the term added via monetary conservativeness to the objective function 'tilts' the optimal choice for b towards 1, which was the optimal choice before considering a conservative objective, and the optimal choice for a towards $-\frac{1}{\xi}$. This proves the claim.

[31] Note that the covariance terms (68) and (69) remain unaffected by treating σ_η^2 as exogenous.

A.6 Proof of lemma 8

From proposition 7 we have $u_t = CB\omega_t$. For $b = 1$ we get (see appendix A.4):

$$CB = \begin{pmatrix} \left(a + \frac{1}{\xi}\right) k & 0 & k \\ (1-k)\left(a - \frac{k}{(1-k)}\frac{1}{\xi}\right) & 0 & -k \\ a & 1 & 0 \end{pmatrix}$$

From equation (64) follows that the variance of the observation noise is

$$\sigma_\eta^2 = \frac{1-k}{k}\frac{1}{1-(1-k)\rho^2}\left(\frac{1}{\xi} + a\right)^2 \sigma_1^2 \tag{76}$$

Therefore, letting $u_t{}' = (u_t^1, u_t^2, u_t^3)'$, the variances of the VAR residuals are

$$Var(u_t^1) = \left(k^2 + k(1-k)\frac{1}{1-(1-k)\rho^2}\right)\left(a + \frac{1}{\xi}\right)^2 \sigma_{v_1}^2$$
$$Var(u_t^2) = \left((1-k)^2\left(a - \frac{k}{(1-k)}\frac{1}{\xi}\right)^2 + (1-k)\frac{1}{1-(1-k)\rho^2}\left(\frac{1}{\xi} + a\right)^2\right)\sigma_{v_1}^2$$
$$Var(u_t^3) = a^2\sigma_{v_1}^2 + \sigma_{v_2}^2$$

From the previous expressions it is obvious that $\frac{\partial Var(u_t^1)}{\partial a} > 0$, $\frac{\partial Var(u_t^3)}{\partial a} > 0$, and straightforward to establish $\frac{\partial Var(u_t^2)}{\partial a} > 0$, provided $a > 0$.

A.7 Proof of lemma 9

Given the policy rule (38), the result is immediate for the variance of the policy instrument. From proposition 7 we have

$$z_t = Dz_{t-1} + CB\omega_t$$

Taking variances on both sides and applying the columnwise vectorization operator $vec(\cdot)$ one gets

$$vec\,(var(z_t)) = (I_{9x9} - D \otimes D)^{-1} vec(CB\Sigma C'B')$$

where $\Sigma = var(\omega_t)$ and I_{9x9} is a 9-dimensional identity matrix. Deriving the explicit expressions for $vec\,(var(z_t))$ and computing the derivatives shows that $\frac{\partial\sigma_p^2}{\partial a} > 0$ and $\frac{\partial\sigma_{y-y_n}^2}{\partial a} > 0$, provided $a > 0$.

A.8 Proof of lemma 10

From the proof of lemma 9 in appendix A.7 one obtains explicit expression for the variance of prices and output gap. Evaluating these at discretionary and at fully optimal policy shows that

$$\frac{\sigma_{p,d}^2}{\sigma_{p,c}^2} = \frac{(1-\rho(1-k))^2(1+\rho(1-k))^2}{(1-\rho^2(1-k))^2}\frac{1}{(1-k)^2}$$
$$\frac{\sigma_{y-y_n,d}^2}{\sigma_{y-y_n,c}^2} = \frac{\left(1+\rho^4-2\rho^2+3k\rho^2-3k\rho^4-2k^2\rho^2+3k^2\rho^4-k^3\rho^4\right)}{(1-\rho^2(1-k))^2}\frac{1}{(1-k)}$$

where $\sigma^2_{p,d}$ and $\sigma^2_{p,c}$ denote the variance of prices under discretionary and commitment policy, respectively, and $\sigma^2_{y-y_n,d}$ and $\sigma^2_{y-y_n,c}$ the corresponding variances of the output gap. As is easily seen from the previous equations, the variance ratios become unbounded as $k \to 1$.

A.9 Proof of lemma 11

Evaluating $\frac{\partial D}{\partial a}$ at the discretionary monetary policy solution delivers

$$\frac{\partial D}{\partial a} = \begin{pmatrix} -\frac{1}{k}\xi\rho\left(-k+1\right)^2 & -\frac{1}{k}\xi\rho\left(-k+1\right)^2 & 0 \\ \frac{1}{k}\xi\rho\left(-k+1\right)^2 & \frac{1}{k}\xi\rho\left(-k+1\right)^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

and shows that for $k \approx 1$ one obtains (45). Moreover, taking the derivative of

$$diag(Var(u_t)) = diag(Var(CB\omega_t))$$

with respect to a delivers

$$\frac{\partial diag(Var(u_t))}{\partial a} = \begin{pmatrix} 2\frac{\left(k^2\rho^2-k\rho^2+1\right)(a\xi+1)\sigma_1^2}{(k\rho^2-\rho^2+1)\xi}k \\ 2\frac{\left(a\xi\rho^2-k\rho^2-a\xi-2ak\xi\rho^2+k^2\rho^2+ak^2\xi\rho^2\right)}{(k\rho^2-\rho^2+1)\xi}\left(k-1\right)\sigma_1^2 \\ 2\sigma_1^2 a \end{pmatrix}$$

Evaluating at the discretionary monetary policy solution and taking the limit $k \to \infty$ delivers (46).